

Markets for Financial Innovation*

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Abstract

Financial securities trade in a wide variety of market structures. This paper develops a theory in which both the market structure of trade and the payoffs of the claims being traded form endogenously. Financial intermediaries use the cash flows of an underlying asset to design securities for investors. The demand for securities arises as investors choose markets then trade using strategies represented by quantity-price schedules. We find that intermediaries create increasingly riskier securities when facing deeper markets in which investors trade more competitively. In turn, investors elicit safer securities when they choose to trade in thinner, more fragmented markets. These findings reveal a novel role for market fragmentation in the creation of safer securities. The model is also informative about which investor classes trade which securities and how the distributional properties of the underlying asset affect the relationship between security design and market structure.

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1 Introduction

It has long been acknowledged that non-financial firms adjust product design in response to market structure.¹ In financial markets, persistent empirical regularities to this effect are lacking. This is especially so when it comes to standardized securities whose payoffs are not commissioned by any one investor. Different standardized securities trade in different venues at any point in time, but historically the same claims have also traded in different market structures. Do financial intermediaries also adjust product design in response to market structure when creating securities to connect investors with markets? If so, what is the relationship between types of claims and the markets in which they trade? Insights into these questions would inform ongoing debates about the desirability of centralized trading and shed light on whether financial regulators can successfully restructure markets and improve efficiency.

To study these issues, we build a tractable model in which both security design and market structure are endogenously determined. We take seriously the fact that standardized securities trade in a wide variety of market structures and the possibility that a wide variety of market structures are supported as equilibria. We explore primitives that shape the relationship between market liquidity and risk and analyze welfare implications of different equilibrium market structures.

Our environment is one where financial intermediaries use the cash flows of an underlying asset to design securities for investors. A security specifies a payoff for every realization of the underlying asset. As in Ross (1976) and Allen and Gale (1994), we consider that financial innovation arises in response to investors' demand. However, key to our model is that the demand for securities is itself endogenous. This demand is modeled in two steps. First, investors choose a market in which to trade. Second, once markets open and investors can trade, their trading strategies are represented by quantity-price schedules, with each investor understanding the impact of her trade on the price of the security.

A distinguishing feature of many investors in financial markets is that their valuations are generally grounded in mean-variance analysis. We capture this most simply with mean-variance preferences. Investors are ex-ante homogeneous but have different ex-post val-

¹See Johnson and Myatt (2003), Johnson and Myatt (2006), and Bar-Isaac, Caruana, and Cuñat (2012).

uations of a security, which allows them to benefit from trading with each other. The idiosyncratic valuations of investors are realized after securities have been designed and an intermediary cannot commit to making payoffs contingent on these valuation shocks. The securities that emerge in equilibrium are thus standardized.

Financial intermediaries are strategic when designing securities, taking into account demand by investors in the markets in which the securities will be traded. Markets can be thinner and more fragmented with investors trading more strategically, or deeper and less fragmented with investors trading more competitively. Investors choose markets understanding that their choices will affect the market structure faced by financial intermediaries and thus the design of the securities that will be traded.

There are two implicit frictions in the environment that are worth making explicit here. First, investors cannot directly invest in the same assets as financial intermediaries. This is realistic as financial intermediaries frequently create asset-backed securities that give investors exposure to markets that they could not otherwise invest in. Mortgage-backed securities are one such example.² Second, intermediaries design securities bounded by limited liability. That is, a security's payoff cannot exceed the payoff of the asset that backs it in any given state of the world. In practice, most securities are implicitly designed to respect this constraint. In our set-up, limited liability is equivalent to the spanning constraint in the financial innovation literature (Duffie and Rahi (1995)) which requires that the securities a financial intermediary issues span the payoff of the asset that backs them.

We obtain two major sets of results. The first set of results characterizes the security that an intermediary finds optimal to offer taking as given the market structure. We show that this security depends monotonically on the depth of the intermediary's market. In particular, we show (i) that the optimal security belongs to the family of debt contracts, paying the lesser of a flat payoff and the full value of the underlying asset in every state of the world, and (ii) that the state in which the security starts paying the flat payoff is higher in markets with more investors. In other words, financial intermediaries design progressively riskier asset-backed securities when facing investors that trade more competitively. In the

²Naturally, there are derivative securities, such as equity options, for which the investors can acquire both the underlying asset (the equity security) and the derivative security (the equity option). These securities are therefore not subject to the first friction in our environment.

limit, the security approaches the payoff of the underlying asset in all states, which we refer to as equity in the spirit of the literature on security design.

The intuition for this first set of results is as follows. When choosing how to design a security, the intermediary's main incentive is to obtain a high price for it. The equilibrium price at which the security is traded is increasing in its mean payoff and decreasing in the variance of its payoffs across states. The intermediary thus faces a trade-off between the mean and the variance of the security he designs, making a debt contract optimal as debt has the least variance among all limited liability securities with the same expected value. Importantly, though, the equilibrium price decreases less with the variance of the security in deeper markets where investors have a lower price impact. Thus, the strength of the mean-variance trade-off faced by the intermediary (and hence where on the spectrum of debt contracts the security lies) depends on the depth of the market. The deeper the market, the less pronounced the trade-off and the more equity-like the intermediary makes his security.

The second set of results focuses on the equilibrium market structure. This is crucial to ensure that the securities intermediaries design in a given market structure can indeed be supported in equilibrium. If no investor benefits from trading in a particular market structure, then we should not expect the corresponding securities to arise in equilibrium.

When choosing which market to trade in, an investor weighs the gains from trade with other investors against the ability to influence the security that the intermediary designs. An investor who trades in a thinner market will have a larger price impact. On one hand, this amplifies the mean-variance trade-off in the intermediary's security design problem and delivers a less risky security. On the other hand, it also amplifies the extent to which the investor will move the price of the security against herself when trading with other investors.

When investors expect to be relatively homogeneous in their valuations of the same security, they anticipate limited benefits from trading with each other and are therefore willing to accept a larger price impact in order to elicit a less variable security from the intermediary. In contrast, when investors expect to be relatively heterogeneous, they understand that they may want to engage in large trades with each other so they seek to limit their price impact by trading in a large market, albeit with a riskier security. Thus, an important

outcome of our model is the following: controlling for the riskiness of the underlying asset, less variable asset-backed securities are traded in thinner, more fragmented markets while more variable asset-backed securities are traded in deeper, more concentrated markets.

To gain further insights into our question, we analyze a simpler version of the model in which asset returns are uniformly distributed. The welfare implications are different for financial intermediaries and investors. If heterogeneity among investors is low, then the symmetric equilibrium that achieves the highest welfare for investors exists in the set of equilibria where debt is traded in thinner, more fragmented markets. In contrast, intermediaries are always better off designing a security for a large market than for a small market. Investors thus benefit at the expense of intermediaries in any equilibrium where debt is traded. In aggregate, however, the benefits to investors in an equilibrium where debt is traded are outweighed by the losses to intermediaries, such that total welfare is higher when markets are deeper, even though the security that emerges in these markets has more variable payoffs.

Lastly, we explore the relationship between security design and market structure in real world markets through the lens of our model. Our findings suggest that institutional investors, who tend to have less dispersion in their preference shocks, are more likely to trade safer securities in fragmented markets. In contrast, retail investors, who tend to be more heterogeneous in their preference shocks, participate in larger markets where they trade riskier securities. We also show that the trading of equity in a centralized market can co-exist with financial intermediaries offering debt securities in fragmented markets, even when securities are backed by the same underlying asset. Other important implications of our model are that the origination of better underlying assets can eliminate the creation of asset-backed securities with less variable payoffs and that the distributional properties of the underlying asset can affect the relationship between market liquidity and the riskiness of asset-backed securities.

Related Literature

This paper relates to several strands of literature. The most relevant studies are those on security design and endogenous market structure.

The literature on security design has been very prolific over recent decades. The classic problem explored in these papers is that of a firm needing to raise funds from an investor to finance an investment project. In exchange, the firm proposes a security to the investor. A common result in this literature is that debt is the optimal security in the presence of asymmetric information or moral hazard (e.g., Gale and Hellwig (1985), Gorton and Pennacchi (1990), Nachman and Noe (1994), DeMarzo and Duffie (1999), Biais and Mariotti (2005), Yang (2017), Hébert (2018), Asriyan and Vanasco (2018)).³ We explore a variant of the typical set-up. In particular, financial intermediaries issue securities which allow investors to have exposure to assets in which they cannot directly invest. The family of debt contracts is optimal even absent informational asymmetries, and, more importantly, financial intermediaries offer low-variance debt only when investors trade in a thin market. As the market gets deeper, the optimal security becomes equity.

Parallel to the literature on security design, there is a body of work on financial innovation that studies the role of security issuances in completing markets. From the seminal paper of Allen and Gale (1991) to the more recent contribution of Carvajal, Rostek, and Weretka (2012), the main focus of this line of research is to analyze whether competition among asset-holders affects their incentives to introduce new securities. Complimentary to this literature, we study a model in which a financial intermediary's decision to issue securities is affected by the strategic competition between investors when trading the securities they are offered.

There is a young but growing literature on endogenous market structure. Babus and Parlatore (2018), Cespa and Vives (2018), Dugast, Üslü, and Weill (2019), Lee and Wang (2018), and Yoon (2018) provide models that seek to explain why trade takes places in a variety of venues, centralized or decentralized. However, in these papers, the asset traded is taken to be exogenous. We endogenize both the security design and the market participation decision, which allows us to study the relationship between the type of security and the market structure in which it is traded.

A small number of papers study the effect of market structure on security design. In a

³In Malenko and Tsoy (2018), a mixture of debt and equity can be optimal when the investor faces Knightian uncertainty about the underlying project's returns. Other models of endogenous capital structure instead assume transaction costs of security issuance as in Allen and Gale (1988); see, for example, Corbae and Quintin (2019) on the cyclical properties of safe debt.

set-up which assumes that investors are better informed about the prospects of the issuer than the issuer himself, Axelson (2007) shows that debt is optimal if the degree of competition among investors is low. Rostek and Yoon (2018) analyze the role of market structure for introducing non-redundant derivatives. In both of these papers, however, the market structure is taken to be exogenous. In our paper, the market structure is endogenously determined. This is important, as it ensures that the securities traded in a given market structure can indeed be supported in equilibrium.

There is a lack of direct empirical work on the joint determination of security design and market structure. However, Biais and Green (2018) provide a thorough documentation of developments in the bond market in the 20th century. They find that when institutions became more important in bond markets, bond markets became thinner. While our model is inherently static (as it develops over three periods), it builds on ingredients that are relevant for investigating these issues. For instance, changes in the dispersion in investors' valuations or in the variance of the underlying asset over time could potentially account for market structure dynamics.

The rest of the paper proceeds as follows: Section 2 introduces the model environment; Section 3 defines and characterizes the equilibrium; Section 4 presents the welfare implications of our model; Section 5 discusses implications of our model for real world markets; Section 6 analyzes some extensions of our model; and Section 7 concludes. All proofs are collected in Appendix A.

2 The Model Set-Up

Our analysis focuses on how financial firms adjust the design of their securities in response to the demand they face from investors. To capture the interactions between investors and financial intermediaries in a simple setting, we adopt a standard security design framework in which we allow investors to trade the security that intermediaries design. To this, we add a market formation stage to capture how investors' demand arises. This is a key step to ensure that the securities intermediaries design in response to investors' demand can indeed be supported in equilibrium.

We consider an economy with three dates, $t = 0, 1, 2$, and two types of agents, financial intermediaries and investors. There are $M \geq 2$ risk neutral, impatient financial intermediaries indexed by $m = 1, \dots, M$. Each intermediary has access to a risky asset Z . The asset stands for loans originated to firms, or mortgages extended to households. We assume that each unit of the asset Z yields a payoff $z(s) \geq 0$ if the aggregate state $s \in [0, S]$ is realized at date $t = 2$. The cumulative distribution function for states is $F(s)$, with $F(\cdot)$ continuous and differentiable, and the probability density function is $f(s)$. Without loss of generality, we assume $z'(\cdot) > 0$.

A market m is associated with each financial intermediary m . In each market m , the intermediary can issue a security W_m that pays $w_m(s)$ in state s at date $t = 2$. For this reason, we can interchangeably refer to the intermediary as the (security) issuer. As in the literature on the spanning role of securities (Duffie and Rahi (1995)), the security payoff is subject to the feasibility constraint

$$w_m(s) \leq z(s), \forall s \in [0, S]. \quad (1)$$

From (1), each unit of W_m is backed by one and only one unit of the asset Z . This is consistent with an interpretation in which each intermediary issues an asset-backed security based on a representative loan that he previously originated. To reduce notation, each intermediary m supplies one unit per capita of the security W_m in his market. In our main specification, we give intermediaries access to a sufficiently large pool of the asset Z so that constraint (1) is satisfied (instead endowing each intermediary with a fixed amount of the asset Z and restricting him to supply a fixed amount of the security W_m can be easily accommodated and does not affect the insights). We then consider an extension where intermediaries optimally choose how many units of Z to acquire, subject to a cost of procuring Z . This is consistent with an interpretation in which each intermediary buys loans from a loan originator and then issues an asset-backed security. Section 6.1 discusses the changes to (1) when intermediaries can choose how many units of Z back each unit of W_m .

There are $N \geq 3$ patient investors, indexed by $i = 1, \dots, N$. Often in financial markets

investor demand for securities is shaped by a mean-variance analysis. We work with the simplest and most tractable specification that captures this, which is mean-variance preferences. Each investor i is also subject to a preference shock θ^i that shifts her marginal utility of consumption, as we describe in detail below. The heterogeneity that θ^i introduces across investors can be interpreted as differences in liquidity needs, in the use of securities as collateral, in technologies to repackage and resell cash flows, or in risk-management constraints, for example. The shock θ^i is independently distributed across investors according to a distribution $G(\cdot)$ with mean μ_θ and standard deviation σ_θ . The realization of the shock θ^i is also independent of the realization of the state s .

Investors do not have access to the asset Z . However, an investor who wants exposure to Z can choose a market m in which she can trade and acquire some quantity of the security W_m that intermediary m designs. In line with Ross (1976) and Allen and Gale (1994), we take the approach that financial innovation is driven by investors' demand. For this, we assume that intermediaries design securities after investors choose markets.⁴ At the same time, our focus is on studying the issuance of standardized securities. For this, we consider that intermediaries design securities before the preference shocks, θ^i , are realized. Thus, a security cannot be customized to address the specific requirements of any particular investor.

We model how investors' demand for securities arises in two steps. First, investors' choices at date $t = 0$ determine a market structure \mathcal{M} . When an investor i chooses a market m , we say that $i \in m$. We denote by n_m the number of investors that choose market m . We consider a market m to be active if and only if $n_m > 2$. In this case, we say that $m \in \mathcal{M}$. A market structure \mathcal{M} is characterized by the number of active markets, M' , and by the number of investors in each market, $\{n_m\}_{m=1}^{M'}$. We define a market structure to be *symmetric* if each active market m has the same number of investors $n_m = n$.

Second, when markets open, we model investors' trading strategies as quantity-price schedules, as in Kyle (1989) and Vives (2011). In particular, the strategy of an investor is a map from her information set to the space of demand functions, as follows. The demand function of an investor $i \in m$ with preference shock θ^i is a continuous function $Q_m^i : \mathbb{R} \rightarrow \mathbb{R}$

⁴In Section 6.2, we discuss the robustness of our findings to an alternative timing in which we allow investors to choose markets after intermediaries design securities.

which maps the price p_m of the security W_m in market m into a quantity q_m^i she wishes to trade

$$Q_m^i(p_m; \theta^i) = q_m^i.$$

An investor i who trades q_m^i units of security W_m in market m at date $t = 1$ consumes C_m^i at date $t = 2$, where

$$c_m^i(s) = q_m^i w_m(s), \tag{2}$$

for each state s .

To summarize, the timing of events is as follows. At date $t = 0$, each investor chooses a market m in which to trade. An investor can choose at most one market. However, multiple investors can choose the same market. Next, the intermediary in market m designs the security W_m . At date $t = 1$, each investor i learns her preference shock θ^i . After this, all markets open and investors in each market m trade the security W_m . At date $t = 2$, the state s is realized. Investors receive payoffs according their final holdings of the security. Each intermediary m pays $w_m(s)$ and receives $z(s)$ per capita. Consumption takes place.

That investors can choose at most one market and that the intermediaries can each design only one security are assumptions we make to ensure tractability. While in reality investors have the opportunity to trade in several markets and intermediaries can offer multiple securities, our set-up is a first step to identify which forces are relevant in the interaction between market structure and security design. Once these forces are understood, many extensions, including but not limited to those in Section 6, are possible.⁵

Given a market structure \mathcal{M} and a security W_m that intermediary m designs at date $t = 0$, the expected payoff of an investor i in market m at date $t = 1$ as she engages in trade is

$$V_m^i = \theta^i E_1(C_m^i) - \frac{\gamma}{2} \mathcal{V}_1(C_m^i) - p_m q_m^i, \tag{3}$$

where $\mathcal{V}(\cdot)$ is the variance operator. We use $E_1(\cdot)$ and $\mathcal{V}_1(\cdot)$ to denote that expectations are being taken over the state s , which is the only unknown at date $t = 1$. The price p_m in

⁵It is worth noting that investors may still choose to trade in one market despite having the opportunity to trade in many markets. For instance, Boyarchenko, Costello, and Shachar (2018) provide evidence that the majority of financial institutions participate in either the corporate bond market or the CDS market, even though there exist bonds and CDSs issued on the same entities.

Eq. (3) is the price at which local market m clears, given that intermediary m supplies n_m units of the security W_m . That is, p_m is such that

$$\sum_{i \in m} Q_m^i(p_m; \theta^i) = n_m. \quad (4)$$

Substituting Eq. (2) into Eq. (3), we obtain that investor i 's objective function at date $t = 1$, before the uncertainty about the state of the world s has been resolved, is

$$V_m^i = [\theta^i E_1(W_m) - p_m] q_m^i - \frac{\gamma}{2} \mathcal{V}_1(W_m) (q_m^i)^2, \quad (5)$$

where $E_1(W_m) \equiv \int_0^S w_m(s) dF(s)$ and $\mathcal{V}_1(W_m) \equiv \int_0^S [w_m(s) - E_1(W_m)]^2 dF(s)$. In this reformulation, the preference shock θ^i captures investor i 's valuation of the payoff she expects to obtain from one unit of the security W_m .

An intermediary m supplies n_m units of the security W_m that he designs in market m , and he receives the price p_m per unit of the security. Aside from designing the security and supplying it to the market, the intermediary is not directly involved in the trade between investors at date $t = 1$. Given a market structure \mathcal{M} and a security W_m that the intermediary designs in a market m with n_m investors at date $t = 0$, intermediary m 's expected payoff at date $t = 1$ is

$$V_m = [p_m + \beta E_1(Z - W_m)] \times n_m,$$

where $\beta \in [0, 1]$ is a discount factor that captures the impatience of intermediaries relative to investors.

The trading protocol through which investors in market m acquire the security W_m corresponds to a share auction as described by Wilson (1979). In particular, our set-up is consistent with the interpretation that each intermediary m places the security W_m with investors in his market by running the following auction. Each investor i in market m is a bidder that submits a schedule indicating the quantity of the security she demands at each price. The supply of the security is perfectly divisible and, in each market m , the security is allocated at the clearing price, p_m , which is the solution to the market clearing condition (4). Each investor i receives a share q_m^i of the security for which she pays $p_m q_m^i$.

3 Equilibrium

In this section, we define and characterize the equilibrium. We start by solving for the trading equilibrium in each market m at date $t = 1$, given a market structure \mathcal{M} and the securities W_m that intermediaries design at date $t = 0$. We then characterize the security that each intermediary designs in equilibrium for his market m at date $t = 0$, given a market structure \mathcal{M} . Lastly, we analyze the market formation game which determines the equilibrium market structure \mathcal{M} at $t = 0$.

Definition 1 *A subgame perfect equilibrium is a market structure \mathcal{M} , a set of securities $\{W_m\}_{m \in \mathcal{M}}$, and a set of demand functions $\{Q_m^i\}_{i \in m}$ for investors in each active market m such that:*

1. Q_m^i solves each investor i 's problem at date $t = 1$

$$\max_{Q_m^i} \left\{ [\theta^i E_1(W_m) - p_m] Q_m^i(p_m; \theta^i) - \frac{\gamma}{2} (Q_m^i(p_m; \theta^i))^2 \mathcal{V}_1(W_m) \right\}; \quad (6)$$

2. W_m solves each financial intermediary m 's problem at date $t = 0$

$$\max_{W_m} \{E_0(p_m) + \beta [E_1(Z) - E_1(W_m)]\} \times n_m, \quad (7)$$

subject to the feasibility constraint (1);

3. *No investor i benefits from deviating and joining a different local market at date $t = 0$, i.e. the expected payoff an investor receives in market m is at least as large as the expected payoff from deviating to market m'*

$$E_0(V_m^i) \geq E_0(V_{m'}^i) \text{ for all } i \in m \text{ and all } m' \neq m. \quad (8)$$

Our notion of equilibrium market structure, described in the third bullet of Definition 1, is related to the concept of pairwise stability introduced in Jackson and Wolinsky (1996), with the difference that we allow for deviations to be unilateral.

It is important to note that all agents act strategically. This implies that each investor $i \in m$ takes into account her price impact in market m when submitting her demand. Similarly, an intermediary understands how the security he designs at date $t = 0$ affects the price at which investors trade it at date $t = 1$. At the market formation stage, each investor also takes into account how her market choice shapes the security that the intermediaries design, as well as the price at which trade takes place at date $t = 1$. To streamline the exposition, we restrict our attention to equilibria in which the market structure is symmetric, intermediaries design the same security, and agents have linear trading strategies.

The rest of this section characterizes the equilibrium. As mentioned earlier, we solve first for the trading equilibrium conditional on a market structure and a set of securities (Section 3.1), then for the equilibrium security conditional on a market structure (Section 3.2), and finally for the equilibrium market structure (Section 3.3).

3.1 The Trading Equilibrium

At date $t = 1$, after each investor i learns her preference shock θ^i , all active markets open and trade takes place. In each market m , an investor chooses her trading strategy in order to maximize her expected payoff, understanding that she has impact on the price p_m . As is standard in similar models, we simplify the optimization problem (6), which is defined over a function space, to finding the functions $Q_m^i(p_m; \theta^i)$ pointwise. For this, we fix a realization of the set of preference shocks, $\{\theta^i\}_{i=1}^N$. Then, we solve for the optimal quantity q_m^i that each investor $i \in m$ demands in market m when she takes as given the demand functions of the other investors in market m . Thus, we obtain investor i 's best response quantity q_m^i in market m for each realization of the preference shocks of the other investors in market m . This gives us a map from prices to quantities, or the investor's optimal demand function point by point. We describe the procedure in detail below.

The first order condition for an investor i in market m is

$$\theta^i E_1(W_m) - p_m - \left(\frac{\partial p_{m,-i}}{\partial q_m^i} + \gamma \mathcal{V}_1(W_m) \right) q_m^i = 0, \quad (9)$$

where $p_{m,-i}$ is the residual inverse demand of investor i implied by

$$q_m^i + \sum_{j \in m, j \neq i} Q_m^j(p_m; \theta^j) = n_m. \quad (10)$$

An investor $i \in m$ chooses to trade a quantity q_m^i of the security W_m so that her marginal benefit equalizes her marginal cost of trading. The first term in the first order condition (9) is the marginal benefit of increasing the final holdings of the security W_m for investor i , which is given by the expected value of the security scaled by the investor's preference shock θ^i . The remaining terms in Eq. (9) represent investor i 's marginal cost of increasing her demand. The second term represents the price that the investor pays to acquire one unit of the security W_m . Investors also incur indirect costs, captured in the last term in Eq. (9). First, since the investors trade strategically, increasing the quantity demanded has an impact on the market clearing price. Second, investors are risk averse, which maps into a holding cost of the security that increases proportionally to the variance of W_m as the quantity demanded increases. The following proposition characterizes the trading equilibrium in a market m .

Proposition 1 *Given a market structure \mathcal{M} and a set of securities $\{W_m\}_{m \in \mathcal{M}}$, there exists a unique symmetric linear equilibrium that characterizes investors' trading strategies in each market m , as follows. The equilibrium demand function of an investor i in market m is*

$$Q_m^i(p_m; \theta^i) = \frac{1}{(1 + \lambda_m) \gamma \mathcal{V}_1(W_m)} [\theta^i E_1(W_m) - p_m], \quad (11)$$

where $\lambda_m^{-1} \equiv (n_m - 2)$ is an index of market depth. The equilibrium price in market m is

$$p_m = \left(\frac{1}{n_m} \sum_{i \in m} \theta^i \right) E_1(W_m) - (1 + \lambda_m) \gamma \mathcal{V}_1(W_m). \quad (12)$$

Proposition 1 shows that investor i buys or sells the security W_m depending on whether her valuation $\theta^i E_1(W_m)$ of the security's expected payoff is above or below the price p_m at which she can trade. However, as can be seen from the denominator of Eq. (11), the investor will restrict the size of her trade for two reasons. First, she is risk averse and the

security is risky. Thus, the more risk averse the investor is (as proxied by a higher γ), the less she will trade. Similarly, the more risky the security is (as reflected in a higher variance of payoffs across states), the less of it the investor trades, everything else constant. Second, the investor has a price impact, $\partial p_{m,-i}/\partial q_m^i = \lambda_m \gamma \mathcal{V}_1(W_m)$, that decreases with market depth, λ_m^{-1} . In other words, the larger the market is, the more the investor can trade without moving the price against herself.

The equilibrium price in market m , characterized by Eq. (12), is the expected payoff of the security W_m , scaled by the average valuation of the investors in market m , minus a risk premium. The risk premium exists because investors are risk averse and, in expectation, have to hold one unit of a risky security. Indeed, it is easy to check that the expected traded quantity is $E_0(q_m^i) = 1$ for any $i \in m$.

Given a realization of investors' preference shocks, $\{\theta^i\}_{i=1}^N$, it follows from Eq. (12) that the price of the security W_m is lower in a thinner market. The price of the security also decreases with the variance of the security, everything else constant. However, the price decreases less with the variance of the security as the market becomes deeper.⁶ These effects arise because investors are strategic and dislike risk. In a smaller market, changes in the demand of an individual investor have a larger impact on the price of the security. Furthermore, the riskier the security is, the less of it a risk averse investor will demand. If an investor demands less of the security, more will be available to other investors. The price will then have to fall so that, on average, other investors are content with holding more of the security. As the size of the market increases, the price impact of any one investor falls. An increase in riskiness is thus met with a smaller decrease in price compared to a smaller market where a strategic decrease in demand by one investor leads to a bigger price drop.

The effects of market depth and the variance of the security on the price are typical of models in which investors strategically trade risky assets in positive net supply by submitting demand functions. In contrast to standard models, however, in our model both the variance of the security and the market depth are endogenous. In particular, the security is the choice of the intermediaries, while the market structure, and implicitly the market depth, is the

⁶To verify this, consider the cross-partial derivative of the price p_m with respect to the variance of the security W_m and the number of investors in market m , holding everything else constant. This derivative is given by $\frac{\partial}{\partial n_m} \frac{\partial p_m}{\partial \mathcal{V}_1(W_m)} \Big|_{E_1(W_m)=cst} = -\frac{\partial \lambda_m}{\partial n_m} > 0$.

outcome of investors' choices. Our paper seeks to understand how these forces interact.

3.2 The Equilibrium Security

At the end of date $t = 0$, after the market structure is determined, each active intermediary m designs a security W_m in response to investors' demand in his market. In particular, an intermediary chooses the payoff $w_m(s)$ of the security for each state s to maximize his expected profit in (7), subject to the feasibility constraint (1). The constraint (1) restricts the intermediary to offer investors a security with a payoff that does not exceed what the intermediary realizes on the asset Z in any state s . Alternatively, since the intermediary is the residual claimant on the payoff of the asset Z , he is effectively designing two securities: one that he offers to investors and one that he keeps for himself. Thus, the constraint (1) simply requires that the two securities exhaust the returns to intermediary m 's asset, as is commonly assumed in the financial innovation spanning literature.

Taking the expectation at date $t = 0$ of the price p_m at which investors in market m trade the security W_m (i.e., the price in Eq. (12)) and substituting it into (7), we obtain that intermediary m designs the security W_m to maximize the following objective function:

$$E_0(V_m) = [\beta E_1(Z) + (\mu_\theta - \beta) E_1(W_m) - (1 + \lambda_m) \gamma \mathcal{V}_1(W_m)] \times n_m. \quad (13)$$

It is transparent that the intermediary benefits from offering a security that pays well in expectation, as the expected price at which investors trade is increasing in $E_1(W_m)$.⁷ At the same time, the intermediary increases his expected profit if he offers a security with low variance, as the expected price at which investors trade is decreasing in $\mathcal{V}_1(W_m)$. In fact, if he were unconstrained, the intermediary would offer a security with infinite mean and zero variance. However, because the payoff of the security W_m cannot exceed the payoff of the asset Z , the intermediary faces a trade-off between the mean and the variance of the security he designs. Since the weight on the variance in the intermediary's expected profit in Eq. (13) depends on the depth λ_m^{-1} of the market in which the security is traded, how exactly this trade-off is resolved will depend on the market structure.

⁷By the law of iterated expectations, $E_1(W_m) = E_0(W_m)$.

Proposition 2 *Suppose $\mu_\theta > \beta$ so that intermediaries find it profitable to design securities for investors. In any market m with n_m investors, intermediary m designs a security W_m with payoffs*

$$w_m(s) = \begin{cases} z(s) & \text{if } s < \bar{s}_m \\ E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} & \text{if } s \geq \bar{s}_m \end{cases} \quad (14)$$

where the threshold state $\bar{s}_m \in [0, S]$ is defined by

$$\bar{s}_m = \begin{cases} z^{-1} \left(E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} \right), & \forall n_m < n_S \\ S, & \forall n_m \geq n_S \end{cases} \quad (15)$$

with n_S finite if and only if the equation

$$\frac{n_S - 2}{n_S - 1} = \frac{2\gamma}{\mu_\theta - \beta} [z(S) - E_1(Z)] \quad (16)$$

has a solution $n_S \geq 3$.

Proposition 2 shows that intermediary m finds it optimal to design a security that will pay the lesser of a flat payoff and the full value of the asset Z in every state of the world. The security payoff depends on the market structure, the distribution of the underlying asset Z , and the preferences of investors and intermediaries. We say that the security is debt if it pays the flat payoff in at least some states (i.e., the security is debt if $\bar{s}_m < S$). The flat payoff that is paid in states $s \geq \bar{s}_m$ represents the face value of the security. If the security replicates the payoff of the asset Z in all states, then the intermediary sells everything to the investors and passes through the payoffs of the underlying asset Z . For convenience, we refer to the security that replicates the payoff of the asset Z in all states as equity.⁸ In our model, equity is the limiting case of a debt security where the threshold state above which the security pays a flat payoff is $\bar{s}_m = S$.

We have the following cases from Proposition 2. If $\frac{2\gamma}{\mu_\theta - \beta} [z(S) - E_1(Z)] \leq \frac{1}{2}$, then the intermediary finds it optimal to sell everything and offer equity in any market structure. If $\frac{2\gamma}{\mu_\theta - \beta} [z(S) - E_1(Z)] \geq 1$, then the intermediary finds it optimal to design a debt security

⁸Typically, in the literature on security design, an equity security has a payoff that yields a fraction of the underlying asset. We extend this definition to accommodate a fraction of 1.

in any market structure, including in markets with infinitely many investors. These two cases represent corner solutions of the intermediary's optimization problem. If instead $\frac{2\gamma}{\mu_\theta - \beta} [z(S) - E_1(Z)] \in (\frac{1}{2}, 1)$, then intermediary m offers a debt security if the number of investors n_m in market m is below a threshold n_S , otherwise he offers equity.

We provide the intuition for why debt is the security that the intermediary chooses from the set of all possible security profiles. A debt security has the following property: there are no two states, s' and s'' , such that $w_m(s') < z(s')$ and $w_m(s') < w_m(s'')$. In other words, if the constraint (1) does not bind in either state s' or state s'' , then the security yields the same payoff in both states, and, if the constraint (1) binds only in one of the two states, the payoff in that state must be smaller than in the flat part of the debt contract. Suppose intermediary m chooses a security that does not have this property. Then a deviation which increases the payoff of the security in state s' by $\varepsilon_{s'} > 0$ and decreases the payoff of the security in state s'' by $\varepsilon_{s''} = \frac{f(s')}{f(s'')}\varepsilon_{s'}$ decreases the variance of the security without changing its mean. Since the intermediary's expected profit in Eq. (13) is decreasing in the variance of the security, it follows that such a deviation is profitable. Therefore, it cannot be optimal for the intermediary to choose any security other than debt. This argument is similar to the one Hébert (2018) uses to show that debt is the optimal contract in the presence of moral hazard. Novel to our framework, however, is how the equilibrium security depends on the market structure in which it is traded. The following proposition characterizes the relationship between the market structure and the debt contract that the intermediary chooses.

Proposition 3 *Suppose that Eq. (16) has a finite solution $n_S \geq 3$. The threshold state \bar{s}_m defined by (15) is increasing in the number of investors n_m in market m as long as $n_m \leq n_S$.*

Proposition 3 shows that when the intermediary designs a debt security, he will adjust its payoff depending on the market in which the security is traded. In particular, the lowest state in which a security W_m pays the flat payoff increases with the number of investors in market m . In other words, conditional on designing a debt security, the intermediary offers a higher face value in a larger market. At the same time, the larger the market, the

more variable the security that the intermediary designs. This property of the equilibrium security extends automatically to the case when Eq. (16) does not have a finite solution and the intermediary offers debt in markets of any size.

To understand Proposition 3, we appeal to the intuition developed at the end of Section 3.1 about the forces that affect the price of a security W_m . To start, consider a state s where the security that intermediary m designs pays $w_m(s) < z(s)$. If the intermediary increases $w_m(s)$ slightly, holding constant the payoffs in all other states, then he increases both the mean and the variance of the security W_m . The increase in the mean of the security works in favor of the intermediary because it increases the price he expects to receive, whereas the increase in the variance of the security decreases the intermediary's expected profit. However, as we explained in Section 3.1, a higher variance has a greater impact on the expected price in a small market than in a large market. In contrast, as we can see from Eq. (13), the impact of a higher mean on the expected price does not depend on the size of the market. Therefore, the marginal benefit to the intermediary of an increase in $w_m(s)$ is independent of n_m , while the marginal cost is decreasing in n_m . Since a profit-maximizing intermediary sets $w_m(s)$ to equate marginal benefit and marginal cost, it follows that he will increase $w_m(s)$ by more in a large market than in a small market. Given that the intermediary finds it optimal to issue a debt security, he can accomplish this by increasing the threshold state above which the security pays a flat payoff. The next corollary formalizes this discussion and follows immediately from Proposition 3.

Corollary 1 *Suppose that Eq. (16) has a finite solution $n_S \geq 3$. The security W_m that the intermediary designs in market m has the following properties:*

1. $\frac{\partial E_1(W_m)}{\partial n_m} > 0$ for any $n_m \leq n_S$;
2. $\frac{\partial V_1(W_m)}{\partial n_m} > 0$ for any $n_m \leq n_S$.

Two polar securities can be of interest: riskless debt, which is a security that has a flat payoff in all states of the world, and equity, which replicates the payoff of the asset Z in every state. Proposition 2 allows us to understand whether these securities can be offered by intermediaries in equilibrium. The results are collected in the following corollary.

Corollary 2 *Fix a market structure \mathcal{M} .*

1. *In any market $m \in \mathcal{M}$ with $n_m \geq n_S$ investors, where $n_S \in [3, \infty)$ and satisfies Eq. (16), the intermediary offers a security that pays the payoff of the asset Z in every state.*
2. *There is no market $m \in \mathcal{M}$ in which the intermediary offers a security that pays a flat payoff in all states of the world.*

The first part of Corollary 2 is a direct implication of Proposition 2 and the discussion that follows it. Any intermediary with at least n_S investors will find it optimal to sell everything and offer equity. The second part of Corollary 2 says that intermediaries will never offer riskless debt. Suppose to the contrary that there is a market size $n_m \geq 3$ for which an intermediary would find it optimal to offer riskless debt. The variance of riskless debt is zero so, from Eq. (13), it must be the case that the intermediary finds it optimal to offer riskless debt for any market size, including in markets with at least n_S investors. This contradicts the first part of Corollary 2, hence the intermediary never finds it optimal to offer riskless debt.

The results in this section characterize the security that an intermediary chooses to design, taking as given the market structure. However, to show that a security can indeed be supported in equilibrium, we need to verify that the market structure in which it trades is also supported in equilibrium. We address this question in the next section.

3.3 The Equilibrium Market Structure

The goal in this section is to analyze whether there exist equilibrium market structures in which the securities that intermediaries design can be traded. We focus on symmetric market structures. In particular, we characterize market structures where each active market m has the same number of investors $n_m = n$ and no investor has an incentive to deviate to a different market at date $t = 0$. We discuss asymmetric equilibrium market structures in Section 6.3.

To understand the incentives of investor i at date $t = 0$ when she chooses a market in which to trade, we need to first evaluate her expected payoff $E_0(V_m^i)$ from being in market

m , given a market structure \mathcal{M} . Substituting the equilibrium demand function $Q_m^i(p_m; \theta^i)$ from Eq. (11) and the equilibrium price p_m from Eq. (12) into the expression for V_m^i in Eq. (5) then taking expectations at date $t = 0$, before the realization of θ^i is known, we obtain

$$E_0(V_m^i) = \frac{\sigma_\theta^2 n_m - 1}{2\gamma n_m} \left(1 - \frac{1}{(1 + \lambda_m^{-1})^2}\right) \frac{[E_1(W_m)]^2}{\mathcal{V}_1(W_m)} + \frac{\gamma}{2} \left(1 + \frac{1}{\lambda_m^{-1}}\right)^2 \left(1 - \frac{1}{(1 + \lambda_m^{-1})^2}\right) \mathcal{V}_1(W_m).$$

If we further substitute the market depth index $\lambda_m^{-1} = n_m - 2$, investor i 's expected payoff becomes

$$E_0(V_m^i) = \frac{\sigma_\theta^2 n_m - 2}{2\gamma n_m - 1} \frac{[E_1(W_m)]^2}{\mathcal{V}_1(W_m)} + \frac{\gamma}{2} \frac{n_m}{n_m - 2} \mathcal{V}_1(W_m). \quad (17)$$

The expected payoff at date $t = 0$ of an investor who will trade the security W_m at date $t = 1$ in a market with n_m investors has two components. The first term in Eq. (17) is proportional to the variance of the investors' preference shocks, σ_θ^2 , and captures the gains from trade with other investors. The larger σ_θ^2 is, the more heterogeneous investors are in how they value the mean payoff of the same security, and the more they benefit from trading with each other. In fact, when σ_θ^2 is small, investors are very similar in their valuation of the security and the equilibrium holdings of each investor approaches 1, which is the per capita supply offered by the intermediary in market m . In this case, an investor's payoff is mainly driven by the risk premium that she commands as compensation for holding a risky security. The second term in Eq. (17) captures the part of the investor's expected payoff that comes from this compensation for risk.

Both the gains from trade and the compensation for risk depend on the depth of the market in which the investor trades. For a given security W_m , the gains from trade term in Eq. (17) increases with n_m , both because the fundamental gains from trade, $\frac{n_m - 1}{n_m} \sigma_\theta^2$, are increasing in the number of market participants (even though the asset supply scales up linearly with the size of the market) and because the price impact of an investor is smaller in a larger market. In contrast, the compensation for risk term is decreasing in n_m , for a given security W_m , because the investor's price impact falls with the size of the market. The security that intermediary m finds optimal to offer (see Proposition 2) also changes with n_m , affecting both terms in Eq. (17) through W_m . Investor i in market m weighs all of these effects at date $t = 0$ when deciding whether to deviate from market

m , which has $(n - 1)$ other investors, to a deeper market m' , which has n other investors. The following proposition provides sufficient conditions for the existence of an equilibrium market structure.

Proposition 4 *Suppose that the asset Z satisfies $\frac{z^{(k)} - E_1(Z|s \leq k)}{\sqrt{\mathcal{V}_1(Z|s \leq k)}} < \sqrt{2}, \forall k \in (0, S]$. Consider all $n \in [3, N]$ such that there exist integers $M_1 \in \mathbb{N}^+$ and $M_2 \in \mathbb{N}^0$ solving*

$$M_1 \times n + M_2 \times (n + 1) = N, \tag{18}$$

with $M_1 + M_2 \leq M$. Then, there exists a scalar $\bar{\sigma} > 0$ such that:

1. For any $\sigma_\theta^2 \leq \bar{\sigma}$, any market structure with M_1 intermediaries each getting n investors and M_2 intermediaries each getting $n + 1$ investors is stable;
2. For any $\sigma_\theta^2 > \bar{\sigma}$, there is at least one stable market structure with one active intermediary and all investors trading in the same market ($n = N$).

As explained above, we focus on equilibria in which the market structure is symmetric. However, given a total number of investors N , a symmetric market structure in which each active market contains n investors may not exist for every value of n . To address the non-divisibility of investors, Proposition 4 extends the definition of a symmetric market structure to allow for a distribution of investors across markets such that there are n investors in some markets and $(n + 1)$ investors in others. Condition (18) specifies when such generalized symmetric market structures exist.⁹

Proposition 4 shows that a variety of symmetric market structures can be supported in equilibrium. This is important, as it informs us that a rich set of securities can be observed in equilibrium. Using Proposition 2, we can infer that, when $\sigma_\theta^2 \leq \bar{\sigma}$, intermediaries offer a debt security if $n < n_S$ and sell everything, i.e., offer equity, if $n \geq n_S$. Similarly, when $\sigma_\theta^2 > \bar{\sigma}$, the intermediary in the equilibrium with a single active market offers equity unless $N < n_S$. We study asymmetric equilibria, and hence the co-existence of different types of securities, in Section 6.3.

⁹Consider $N = 100$. A market structure in which there are $n = 8$ investors in each market does not exist, as it would require a fractional number of intermediaries. However, there exists a market structure in which there are $M_1 = 8$ markets each with $n = 8$ investors and $M_2 = 4$ markets each with $n = 9$ investors.

The multiplicity of security profiles that can be sustained in equilibrium when $\sigma_\theta^2 \leq \bar{\sigma}$ is consistent with the diverse universe of claims that investors can trade in financial markets. In practice, we observe a wide variety of market structures across time and space, and historically the same claims traded in both centralized and decentralized markets. The multiplicity of equilibria in our model arises when investors prefer to trade less variable securities. In this case, if a symmetric market structure with M intermediaries is an equilibrium, then a symmetric market structure with fewer than M intermediaries is also an equilibrium. The multiplicity of equilibria would collapse if investors in different markets could coordinate to move together to a new market or if each investor had to provide consent for others to join her market. In either scenario, only the most fragmented market structure that achieves the global maximum of the investor value function would be supported in equilibrium. Our assumption about lack of consent is grounded in features of many real-world financial markets. That investors cannot coordinate is also standard, e.g., in the literature on trade in OTC markets initiated by Duffie, Gârleanu, and Pedersen (2005), which precludes the possibility that agents can coordinate to alleviate search frictions.

In words, the results in Proposition 2 and 4 can be synthesized as follows. When investors' demand is fragmented, as is the case when investors are relatively homogenous, financial intermediaries respond by designing debt securities. In consequence, debt securities are traded in a larger number of smaller, less liquid, markets.¹⁰ When investors' demand is instead consolidated, as is the case when investors are relatively heterogeneous, financial intermediaries respond by passing through the payoff of the underlying asset. Thus, equity securities are traded in a smaller number of larger, more liquid, markets. Another, consistent implication is that financial intermediaries that issue equity-like securities have a higher market share than intermediaries that issue debt-like securities backed by the same underlying asset. Note that in our model, by construction, the equilibrium security is always backed by the same underlying asset regardless of whether trade occurs in a more fragmented or a more concentrated market structure.

Proposition 4 shows that the variance of investor preference shocks, σ_θ^2 , helps determine which market structures can be supported in equilibrium. When σ_θ^2 is small, investors

¹⁰A market is liquid if the security can be traded with little impact on its price. We discuss this more formally in Section 5.

will not differ much in their valuations of the same security. The gains from trade are therefore low and investors anticipate that they will trade little with each other. Given this, investors are willing to trade in smaller markets, where they can use their larger price impact to obtain from intermediaries a less variable security whose remaining risk is well compensated. While the larger price impact also hurts the investor when she trades the security with other investors in the same market, this concern is muted because she anticipates trading little with other investors. In contrast, when σ_θ^2 is large, the gains from trade are also large. Investors understand that they may want to make large trades with each other in order to reap these gains, hence they seek to minimize their price impact by trading in a large market, albeit with a riskier security.

It is important to notice that investors' preferences shape the payoffs of the security traded in equilibrium both directly and indirectly. First, because the expected price at which a security W_m trades is increasing in the mean μ_θ of the investor preference shocks, μ_θ directly enters the optimization problem of intermediary m and thus directly affects the payoffs of the security that he finds optimal to design. Second, although the variance of the investor preference shocks does not appear directly in the payoffs of the security derived in Proposition 2, σ_θ^2 plays an important role in determining which securities are traded in equilibrium. The payoffs of the equilibrium security in market m depend directly on the number of investors n_m , and σ_θ^2 affects an investor's decision about which market to trade in. Thus, as we discussed above, when σ_θ^2 is high, investors value trading in deeper markets, which induces the intermediary to offer riskier securities, while, when σ_θ^2 is low, investors prefer trading in thinner markets, which induces the intermediary to offer less variable securities.

4 Welfare and Profits

In this section, our goal is to gain insights into welfare and expected profits by exploring some simple examples. In particular, we are interested in which equilibrium market structure yields the highest welfare for investors, which equilibrium yields the highest welfare for intermediaries, and whether any of the equilibria coincide with the solution to a social

planning problem.

As in Proposition 4, we consider equilibrium market structures with M_1 intermediaries each getting n investors and M_2 intermediaries each getting $(n + 1)$ investors, such that condition (18) is satisfied. In each active market, an investor obtains an expected profit $E_0(V_m^i)$ given by Eq. (17), while the intermediary receives an expected profit $E_0(V_m)$ given by Eq. (13). Aggregate welfare can then be defined as

$$\begin{aligned} \mathcal{W} = & n \times M_1 \times E_0(V_m^i|_{n_m=n}) + (n + 1) \times M_2 \times E_0(V_m^i|_{n_m=n+1}) \\ & + M_1 \times E_0(V_m|_{n_m=n}) + M_2 \times E_0(V_m|_{n_m=n+1}). \end{aligned}$$

To understand which driving forces determine aggregate welfare, it is useful to review the profits of investors and intermediaries, paying special attention to how they depend on the depth of the market.

Eq. (5) gives the profit of investor i after her preference shock θ^i is realized but before the state s is known. Evaluated at the equilibrium demand function $Q_m^i(p_m; \theta^i)$ derived in Proposition 1, Eq. (5) simplifies to

$$V_m^i = \frac{1 + 2\lambda_m}{2\gamma(1 + \lambda_m)^2} \frac{[\theta^i E_1(W_m) - p_m]^2}{\mathcal{V}_1(W_m)}. \quad (19)$$

Given a market depth λ_m^{-1} (in essence, a market size n_m) and a security price p_m , Eq. (19) implies that, among all securities with the same mean payoff $E_1(W_m)$, investor i would prefer the security with the least variance $\mathcal{V}_1(W_m)$. In other words, investor i would prefer debt.

Compare this to Eq. (17), which represents investor i 's expected profit when Eq. (5) is evaluated at both the equilibrium demand function $Q_m^i(p_m; \theta^i)$ and the equilibrium price p_m derived in Proposition 1. Given a market depth λ_m^{-1} , Eq. (17) implies that, among all securities with the same mean payoff $E_1(W_m)$, an investor i who takes into account her price impact would only prefer the security with the least variance $\mathcal{V}_1(W_m)$ if she expects investors to have very disperse valuations. In other words, investor i would prefer debt if σ_θ^2 is high but equity if σ_θ^2 is low.

The endogeneity of market depth and its effect on security design reverses this preference. A key feature of the equilibrium in our model is that investors take into account not only that they have a price impact when they trade but also that their market choice affects market depth and hence the payoffs of the securities that intermediaries design. Thus, an investor's expected profit in Eq. (17) depends on the depth of the market both directly and indirectly, with the indirect effect coming through the equilibrium security W_m derived in Proposition 2. The two terms in Eq. (17) – the gains from trade term and the compensation for risk term – can move in opposite directions as the market becomes deeper. For any underlying asset Z satisfying the sufficient conditions on $z(\cdot)$ in Proposition 4, the compensation for risk term is decreasing in n_m when evaluated at the equilibrium security.¹¹ In contrast, the gains from trade term is potentially increasing in n_m , as shown in the next proposition.

Proposition 5 *Consider an asset Z with payoffs $z(s) = z(0) + s^\alpha$, where $z(0) \geq 0$ and $\alpha > 0$. Suppose that the state s is uniformly distributed according to $f(\cdot) = \frac{1}{S}$. Evaluating an investor's expected profit in Eq. (17) at the equilibrium security $W_m^* \equiv W(n_m)$ derived in Proposition 2, an increase in n_m :*

1. *Increases the gains from trade term,*

$$G(n_m) \equiv \frac{\sigma_\theta^2 n_m - 2 [E_1(W(n_m))]^2}{2\gamma n_m - 1 \mathcal{V}_1(W(n_m))},$$

for all $n_m \geq 3$ if $z(0)$ is not too large;

2. *Decreases the compensation for risk term,*

$$R(n_m) \equiv \frac{\gamma}{2} \frac{n_m}{n_m - 2} \mathcal{V}_1(W(n_m)),$$

for all $n_m \geq 3$ if $\left(\frac{1+\alpha}{4\alpha} \frac{\mu_\theta - \beta}{\gamma S^\alpha}\right)^{\frac{1}{1+\alpha}} > \frac{2\alpha-1}{1+2\alpha}$.

¹¹See the proof of Proposition 4. We emphasize that these conditions on $z(\cdot)$ are sufficient but not necessary. For example, in the class of functions $z(s) = z(0) + s^\alpha$ with a uniformly distributed aggregate state $f(s) = \frac{1}{S}$:

$$\frac{z(k) - E_1(Z|s \leq k)}{\sqrt{\mathcal{V}_1(Z|s \leq k)}} = \sqrt{2\alpha + 1}$$

and, therefore, $\frac{z(k) - E_1(Z|s \leq k)}{\sqrt{\mathcal{V}_1(Z|s \leq k)}} < \sqrt{2}$ if and only if $\alpha < \frac{1}{2}$. However, as shown in Proposition 5, the compensation for risk term is also decreasing in n_m if, for example, $\alpha = 1$ and $\frac{\mu_\theta - \beta}{\gamma S} > \frac{2}{9}$.

Consistent with Proposition 4 and the intuition developed at the end of Section 3.3, Proposition 5 implies that investors will prefer equity if σ_θ^2 is high but debt if σ_θ^2 is low. This constitutes a reversal in the types of securities preferred by investors (as a function of the expected dispersion in their valuations) relative to the case where investors only take into account the price impact of their trades for a given market depth.

Proposition 6 *Consider $z(\cdot)$ and $f(\cdot)$ as in the statement of Proposition 5. Also suppose $N > n_S$ with $n_S \in [3, \infty)$. It follows that:*

1. *An active intermediary's expected profit $E_0(V_m)$ in Eq. (13) is increasing in n_m when W_m is the equilibrium security derived in Proposition 2.*
2. *If $z(0)$ is not too large, then, for any value of σ_θ^2 , the equilibrium in which all investors trade in a single market and the intermediary sells everything, or offers equity, achieves the highest aggregate welfare.*

The first part of Proposition 6 says that an intermediary is always better off designing a security for a large market than for a small market. Investors have less price impact in large markets, so the intermediary is able to command a higher price for whatever security he designs. At the same time, Proposition 5 implies that an investor will be worse off in a large market than in a small market when σ_θ^2 is sufficiently low. Therefore, investors benefit at the expense of intermediaries in any equilibrium where debt is traded. Recall from Proposition 4 that there exist multiple symmetric equilibria when $\sigma_\theta^2 < \bar{\sigma}$. If the variance of investor preference shocks is low enough, the symmetric equilibrium that achieves the highest welfare for investors exists in the set of equilibria where investors trade in many small markets in which financial intermediaries offer debt.

The second part of Proposition 6 says that the benefits to investors of an equilibrium in which intermediaries offer debt are outweighed by the losses to intermediaries, at least in environments where it is impossible to design a security that has high returns in all states of the world (i.e., environments where $z(0)$ is low). First, the expected, per-capita profit of an active intermediary increases more quickly with n_m than the expected profit of an investor decreases with n_m . Second, the non-linear relationship between market size and

the price impact of investors means that total welfare across intermediaries is maximized when there is only one active intermediary.

The results in the second part of Proposition 6 also characterize the solution to a fully constrained social planning problem; that is, the problem of a social planner who chooses a market structure, a set of securities, and a set of demand functions to maximize aggregate welfare subject to the equilibrium conditions in Definition 1. However, as an alternative, we can consider a social planner who: (i) opens M_1 markets each with n investors and M_2 markets each with $n + 1$ investors such that condition (18) holds; (ii) designs a security W_m subject only to the feasibility condition (1); (iii) allocates to investor i in market m a quantity q_m^i of the security W_m after the realization of investor preference shocks, where $\sum_{i \in m} q_m^i = n_m$ for each market m ; and (iv) allocates to the intermediary in each market m a quantity n_m of the security $(Z - W_m)$. The planner in this alternative planning problem still seeks to maximize the aggregate welfare of intermediaries and investors, but he is no longer constrained to choose among solutions that arise as a decentralized equilibrium.

In the alternative planning problem just described, it is straightforward to show that the social planner opens a single market in which all investors trade a zero-variance security (i.e., riskless debt). We omit the proof for brevity, but the intuition is as follows. A security with zero variance neutralizes the risk aversion of the investors. Maximum aggregate welfare is then achieved by allocating unboundedly positive positions q_m^i to investors whose realization of θ^i exceeds the market average and unboundedly negative positions to the rest to satisfy $\sum_{i \in N} q_m^i = N$. If the planner is restricted to design a positive-variance security, then he will open a single market in which investors take large but finite positions on the closest possible security to riskless debt.

The lesson from the alternative planning problem is that the planner can achieve higher welfare by decoupling the security design choice from the market structure choice. In particular, the planner would like to design a debt security for risk averse investors and he would like all investors to trade this security in the same market in order to maximize the gains from trade. The problem is that security design cannot be decoupled from market structure in equilibrium. Intermediaries respond to market-based incentives when designing a security for investors to trade. These incentives come from the price of the security, which

is endogenously less sensitive to investors' risk aversion in a large market because the price impact of an individual investor is decreasing in market size. Thus, when $n_S \in [3, \infty)$, the decentralized equilibrium supports either a fragmented market structure in which financial intermediaries offer debt, or a consolidated, large market, in which a financial intermediary sells the entire payoff of the underlying asset Z .

5 Comparative Statics and Real World Markets

In this section we seek to elucidate the relationship between security design and market structure in real world markets through the lens of our model. We undertake three exercises. First, we study the relationship between the type of security and the liquidity of the market in which it is traded. Second, we explore the relationship between different classes of investors and the securities they are trading. Third, we look at the role of the underlying asset in determining the market structure in which a security is traded. Proofs are collected at the end of Appendix A.

5.1 Liquidity

A natural measure of liquidity in our model is the price impact of an individual investor. Specifically, a market is liquid if the security can be traded with little impact on its price.

Recall from Section 3.1 that the price impact of investor i in market m is $\partial p_{m,-i} / \partial q_m^i = \lambda_m \gamma \mathcal{V}_1(W_m)$, where $\lambda_m^{-1} \equiv (n_m - 2)$ and, in equilibrium, W_m depends on n_m as demonstrated in Proposition 2. Under the conditions stated in Proposition 4, the total derivative of $\partial p_{m,-i} / \partial q_m^i$ with respect to n_m is negative.¹² In other words, a larger market in our model is also a more liquid market. Thus, our model suggests that securities with less variable payoffs, controlling for the riskiness of the underlying asset Z and for investors' preferences, trade in less liquid markets.

Some suggestive evidence in favor of our prediction comes from the work of Friewald, Jankowitsch, and Subrahmanyam (2017). Within the set of mortgage-backed products, they find that mortgage-backed securities (MBS) are much more liquid than collateralized

¹²This follows immediately from the proof of Proposition 4.

mortgage obligations (CMO). Both products derive their cash flows from underlying pools of mortgage loans, but MBS are characterized as “pass-through” securities, meaning that their payoffs will be as variable as the payoffs of the underlying asset. The authors attribute only part of the liquidity difference between MBS and CMO to differences in government guarantees, suggesting an interesting relationship between market liquidity and payoff structure that is consistent with the predictions of our model.

At this stage it is worth noting that primary and secondary markets are one and the same in our model. We believe this to be a reasonable simplification. While in practice we distinguish between trade in primary and secondary markets, these markets are typically tightly linked. In particular, a more liquid secondary market makes the primary market more liquid as well. It would follow from our model that an individual investor has limited price impact against the intermediary in a liquid primary market. Hence, the intermediary can design a more variable security and investors will accept it (i.e., they will not leave the primary market) because the security can be re-traded in a liquid secondary market after the realization of preference shocks. In this way, liquidity of the secondary market supports liquidity of the primary market.

5.2 Investor Classes

Our model also allows us to think about various types of investor classes.

The first dimension on which investors can be grouped into different investor classes is the degree of heterogeneity in their preference shocks. Institutional investors, for example, are likely to have lower σ_θ^2 than retail investors. Our model implies that as σ_θ^2 decreases, more fragmented market structures become stable and these are precisely the market structures that support the issuance of securities with less variable payoffs. Increases in σ_θ^2 have the opposite effect, eliminating market structures that deliver less variable securities. Then, under this interpretation, institutional investors are more likely to trade less variable securities in fragmented markets, while retail investors participate in larger markets where they trade riskier securities. These findings align with the stylized facts documented by Biais and Green (2018) about the 20th century corporate bond market.

Second, we can consider variation in investors’ tolerance for risk. Risk aversion in our

model is captured by the parameter γ , which is (proportional to) the investor's marginal disutility from variance. A decrease in γ increases the importance of the gains from trade term relative to the compensation for risk term in the investor's expected profit. A decrease in γ also directly affects the payoffs of the security derived in Proposition 2, increasing the mean and variance of the equilibrium security for a given market size $n_m \leq n_S$, along with decreasing the threshold market size n_S at which the intermediary offers equity. The same security will thus trade in smaller markets as risk aversion decreases. In practical terms, an increase in investors' risk appetite decreases the liquidity of safer securities, consistent with long-standing practitioner intuition. Whether smaller markets are stable depends on other parameters, including σ_θ^2 , as a decrease in γ amplifies both the rate at which the gains from trade term increases with n_m and the rate at which the compensation for risk term decreases with n_m .

Note that these are predictions based on comparative statics and not an equilibrium analysis of different types of investors trading against each other. However, our model provides a platform to explore such issues, and we leave it for future work.

5.3 The Underlying Asset

One of the main implications of our model is that, controlling for the riskiness of the underlying asset Z , financial intermediaries design progressively riskier asset-backed securities when facing deeper markets. We now explore how equilibrium in our model depends on the characteristics of the underlying (original) asset, including how changes in the riskiness of this asset affect the relationship between market liquidity and the riskiness of the asset-backed security designed by the intermediary.

We consider two exercises with respect to the distributional properties of Z . First, we change only the mean of the distribution from which the payoff of the original asset Z is drawn. Second, we change both the mean and the variance.

To conduct the first exercise, we consider the specification $z(s) = z(0) + s$ for the payoffs of the original asset Z , where $s \in [0, 1]$ is uniformly distributed. The experiment is to increase $z(0)$, which has the effect of increasing the mean payoff of the original asset without changing its variance. For a given market size n_m , the equilibrium security W_m has

the same shape as before (i.e., the mapping from n_m to \bar{s}_m does not change). The variance of W_m also does not change, but the mean payoff $E_1(W_m)$ increases because the payoffs in each state are shifted up by the constant $z(0)$. It remains to check whether the set of stable market structures changes. Consider μ_θ such that Eq. (16) has a finite solution $n_S \geq 3$, which is to say there exists a stable market structure that supports equity. The increase in $z(0)$ has no effect on the compensation for risk term in the investor's expected profit. However, by increasing $E_1(W_m)$ relative to $\mathcal{V}_1(W_m)$ for any market size $n_m \in [3, n_S]$, the increase in $z(0)$ amplifies the rate at which the gains from trade term increases with n_m . For moderate values of σ_θ^2 , this can erode the stability of some fragmented market structures and push towards equilibria where more variable securities are traded. In other words, when there is moderate dispersion in investor preferences, the origination of a better asset Z by financial intermediaries, as captured by a higher mean for the same variance, can eliminate the creation of asset-backed securities with less variable payoffs.¹³

For the second exercise, we consider $z(s) = \kappa s$, where $s \in [0, 1]$ is still uniformly distributed and $\kappa > 0$ is a parameter that affects both the mean and the variance of the original asset Z . Specifically, the higher is κ , the higher are both the mean and the variance of the original asset. For a given market structure, it follows that intermediaries will offer less variable securities as κ decreases. However, when heterogeneity across investors, σ_θ^2 , is low and multiple equilibria are supported, less variable securities can be traded in larger, more liquid, markets as κ changes. In particular, the security designed by an intermediary whose underlying asset is less risky (as captured by lower κ) can be both less variable and traded in a deeper market than the security designed by an intermediary whose underlying asset is more risky (as captured by higher κ). This is consistent with the observation that bonds issued for investment-grade firms are more liquid than those issued for high-yield firms.

¹³For σ_θ^2 sufficiently low, the gains from trade term is of second-order importance in the investor's market choice, and, for σ_θ^2 sufficiently high, fragmented market structures are not stable to begin with, i.e., even at $z(0) = 0$, before the increase in $z(0)$.

6 Robustness

This section considers alternative formulations of our model. We demonstrate that equilibria where investors trade in many small markets in which financial intermediaries offer debt and equilibria where all investors trade in a single market in which the financial intermediary offers equity can be supported in all of these formulations.

6.1 Costly Supply

Up to this point, we have assumed that each intermediary m backs each unit of the security W_m with one and only one unit of Z . This assumption allowed us to abstract from mechanical effects that arise from having a fixed supply of Z in each market, which would only reinforce our results. We can relax this assumption and allow the intermediary to choose how many units of Z back each unit of W_m , subject to a cost of procuring Z . In particular, intermediary m incurs a cost $c(A_m)$ to acquire A_m units of Z which he then uses to back n_m units of W_m . The cost function satisfies the standard conditions $c(0) = 0$ and $c'(\cdot) > 0$. The intermediary now chooses W_m and A_m subject to the feasibility constraint

$$n_m w_m(s) \leq A_m z(s), \forall s \in [0, S].$$

This constraint replaces (1). The rest of the model is as before.

Appendix B shows that the key insights of Propositions 2, 3, and 4 continue to hold. The equilibrium security W_m is a debt security with threshold state $\bar{s}_m \in [0, S]$. As before, \bar{s}_m is increasing in n_m so that the face value of W_m increases and the security becomes more equity-like as the market size increases. All investors trading in a single market in which the financial intermediary offers equity is an equilibrium when the total number of investors N is large. However, for the same N , there also exist equilibria where investors choose to trade in many small markets in which financial intermediaries offer debt if heterogeneity in investor preference shocks, σ_θ^2 , is low.

6.2 Timing

Another assumption in our set-up relates to the timing of events. Specifically, we have assumed that at date $t = 0$ financial intermediaries design securities after investors choose markets. An alternative is that at date $t = 0$ investors choose markets after intermediaries design securities. Then, as before, each investor i learns her preference shock θ^i at date $t = 1$, after which all markets open and investors in each market trade the security that the corresponding intermediary has designed.

Under this alternative timing, investors still make their market choice before the realization of preference shocks, and, hence, financial intermediaries issue standardized securities. However, intermediaries can now compete for investors through security design. When designing the security first, the intermediary commits to a particular payoff profile before investors choose their markets. In other words, the intermediary designs a security whose payoff profile is independent of the number of investors who show up. Nevertheless, the intermediary is rational so the security design problem will take into account the best responses of investors.

We consider two financial intermediaries and study the existence of equilibria in which the market structure is symmetric. As demonstrated in Appendix C, the trading equilibrium is still characterized by Proposition 1. Moreover, we show that a symmetric market structure is supported in equilibrium for σ_θ^2 low and the equilibrium security has the same properties as in our main specification. That is, the security that prevails in equilibrium is debt, and the threshold state above which the security delivers a flat payoff is increasing in the number of investors in each intermediary's market.

Thus, even under the alternative timing considered here, a symmetric equilibrium with two large markets will involve the trading of a more equity-like security than a symmetric equilibrium with two small markets, consistent with the results in our main set-up.

6.3 Asymmetric Equilibrium

We conclude this section by discussing the existence of asymmetric equilibria. The trading equilibrium in Section 3.1 and the intermediary's security design in Section 3.2 were derived for an arbitrary market size, but attention was restricted to symmetric equilibria – that is,

equilibria where all markets were equally sized – when deriving stable market structures in Section 3.3.

We now show that there are equilibria in the class of asymmetric market structures. The investor’s expected profit, $E_0(V^i(n_m))$, in a market of size n_m is still given by Eq. (17), with W_m evaluated at the equilibrium security derived in Proposition 2. We write $E_0(V^i(n_m))$ rather than just $E_0(V_m^i)$ to make explicit the dependence of the investor’s expected profit on n_m , both directly in Eq. (17) and indirectly through the dependence of W_m on n_m in Proposition 2.

Consider an asymmetric market structure with one market of size n_B and $(M' - 1)$ markets of size n_m , where $n_B > n_m + 1$ and $n_B + (M' - 1) \times n_m = N$. This market structure is stable if and only if

$$E_0(V^i(n_B)) > E_0(V^i(n_m + 1))$$

and

$$E_0(V^i(n_m)) > \max\{E_0(V^i(n_m + 1)), E_0(V^i(n_B + 1))\}.$$

In words, no investor in the large market n_B wants to move to a smaller market (i.e., a market that has n_m other investors as opposed to $n_B - 1$ other investors). Similarly, no investor in a small market n_m wants to move to a slightly larger market (i.e., a market that has n_m other investors as opposed to $n_m - 1$ other investors) or to a much larger market (i.e., a market that has n_B other investors). Since there is only one market with n_B investors, it is not possible for one of them to move to an even larger market (i.e., a market that has n_B other investors as opposed to $n_B - 1$), hence we do not need $E_0(V^i(n_B))$ to exceed $E_0(V^i(n_B + 1))$.

To fix ideas, consider the following parameterization: $z(s) = s$ and $f(s) = \frac{1}{S}$ for all $s \in [0, S]$, with $S = 1$, $\frac{\mu\theta - \beta}{\gamma} = 1.25$, and $\frac{\sigma\theta}{\gamma} = 0.275$. This implies $n_S = 6$ in Proposition 2, meaning that a financial intermediary offers equity in any market with six or more investors. It is straightforward to verify that one large market with $n_B = 75$ investors trading equity and any number of small markets each with $n_m = 4$ investors trading debt

is a stable asymmetric equilibrium.¹⁴ Our model therefore admits asymmetric equilibria and, in particular, asymmetric equilibria where some financial intermediaries issuing debt securities and one intermediary issuing equity co-exist, even when securities are backed by the same underlying asset Z .

7 Conclusion

This paper has developed a tractable model of financial innovation to address a critical question: what is the relationship between the types of securities offered and the market structures in which they trade? A central finding of our paper is that financial intermediaries design progressively riskier securities when facing deeper, less fragmented markets in which investors trade more competitively. Market fragmentation thus plays an important role in the creation of safer securities.

The methodological novelty in our paper is that both security design and market structure are endogenously determined. This is important, as it ensures the securities created for a given market structure are indeed supported in equilibrium. Financial intermediaries design asset-backed securities taking into account investors' demand in the markets in which the securities will be traded. Investors choose markets understanding that their choices will affect market depth and thus the design of the securities that will be available for trade.

When choosing how to design a security, an intermediary's main incentive is to obtain a high price for it. As usual, the equilibrium price at which the security is traded is increasing in its mean payoff and decreasing in the variance of its payoffs across states. The intermediary thus faces a trade-off between the mean and the variance of the security he designs, making a debt contract the optimal one. Importantly, we show that the equilibrium price decreases less with the variance of the security in deeper markets where investors have a lower price impact. Thus, the strength of the mean-variance trade-off faced by the intermediary depends on the depth of the market. The deeper the market, the less pronounced the trade-off and the higher the face value of the debt contract offered.

¹⁴The relevant expected profits for an investor are: $E_0(V^i(4)) = 0.3151669\frac{\gamma}{2}$; $E_0(V^i(5)) = 0.3088872\frac{\gamma}{2}$; $E_0(V^i(75)) = 0.3094256\frac{\gamma}{2}$; and $E_0(V^i(76)) = 0.3094356\frac{\gamma}{2}$. Notice $E_0(V^i(75)) > E_0(V^i(5))$ and $E_0(V^i(4)) > \max\{E_0(V^i(5)), E_0(V^i(76))\}$, which are the stability conditions outlined above.

When choosing a market in which to trade, an investor weighs the gains from trade with other investors against the ability to influence the security that the financial intermediary designs. An investor who trades in a thinner, more fragmented market will have a larger price impact. On one hand, this amplifies the mean-variance trade-off in the intermediary's security design problem and delivers a less risky security. On the other hand, it also amplifies the extent to which the investor will move the price of the security against herself when trading with other investors.

As in Dugast, Üslü, and Weill (2019), investors' types play a key role in determining the market structure in which trade occurs. However, in our model, investors' preferences affect directly and indirectly the security that will be traded. When investors expect to be relatively heterogeneous in their valuations of the same security, they understand that they may want to engage in large trades with each other so they seek to limit their price impact by trading in a large market, albeit with a riskier security. In contrast, when investors expect to be relatively homogeneous in their valuations, they anticipate trading little with each other and are thus willing to accept a larger price impact in thinner, more fragmented markets in order to elicit less variable securities from financial intermediaries.

Through the lens of this model, we provide a novel perspective on the relationship between security design and market structure in real world markets. Our findings suggest that institutional investors, who tend to have less dispersion in their preference shocks, are more likely to trade less variable securities in fragmented markets. In contrast, retail investors, who tend to be more heterogeneous in their preference shocks, participate in larger markets where they trade riskier securities. Other important implications of our model are that the origination of better underlying assets can eliminate the creation of asset-backed securities with less variable payoffs and that the distributional properties of the underlying asset can affect the relationship between market liquidity and the riskiness of asset-backed securities. Having developed a parsimonious framework at the intersection of market structure and security design, our model provides a platform on which many extensions can be considered and offers fruitful avenues for future work.

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Online Appendices

Appendix A – Proofs

Proof of Proposition 1

Rearrange the first order condition of investor i in Eq. (9) to isolate:

$$q_m^i = \frac{\theta^i E_1(W_m) - p_m}{\frac{\partial p_{m,-i}}{\partial q_m^i} + \gamma \mathcal{V}_1(W_m)} \quad (\text{A.1})$$

for any $i \in m$. Use this expression to substitute out $Q_m^j(\cdot)$ from Eq. (10) for all investors $j \neq i$ in market m :

$$q_m^i + \sum_{j \in m, j \neq i} \frac{\theta^j E_1(W_m) - p_m}{\frac{\partial p_{m,-j}}{\partial q_m^j} + \gamma \mathcal{V}_1(W_m)} = n_m \quad (\text{A.2})$$

We focus on symmetric linear equilibria in which the price impact $\frac{\partial p_{m,-j}}{\partial q_m^j}$ does not vary across investors within the same market. This permits rearranging Eq. (A.2) to isolate:

$$p_m = \frac{\sum_{j \in m, j \neq i} \theta^j}{n_m - 1} E_1(W_m) - \frac{n_m - q_m^i}{n_m - 1} \left(\frac{\partial p_{m,-j}}{\partial q_m^j} + \gamma \mathcal{V}_1(W_m) \right)$$

which then implies:

$$\frac{\partial p_{m,-i}}{\partial q_m^i} = \frac{1}{n_m - 1} \left(\frac{\partial p_{m,-j}}{\partial q_m^j} + \gamma \mathcal{V}_1(W_m) \right)$$

Invoking symmetry ($\frac{\partial p_{m,-i}}{\partial q_m^i} = \frac{\partial p_{m,-j}}{\partial q_m^j}$), we obtain:

$$\frac{\partial p_{m,-i}}{\partial q_m^i} = \lambda_m \gamma \mathcal{V}_1(W_m) \quad (\text{A.3})$$

where $\lambda_m \equiv \frac{1}{n_m - 2}$. Substituting Eq. (A.3) into Eq. (A.1) delivers the equilibrium demand function $Q_m^i(p_m; \theta^i)$ in Eq. (11). Substituting Eq. (11) into the market clearing condition $\sum_{i \in m} Q_m^i(p_m; \theta^i) = n_m$ then delivers the equilibrium price p_m in Eq. (12). ■

Proof of Proposition 2

Intermediary m designs a security W_m to maximize his expected payoff in Eq. (7), subject to the state-by-state feasibility constraint (1).

Letting $v(s) \geq 0$ denote the Lagrange multiplier on the feasibility constraint for state s , we can write the Lagrangian for intermediary m 's optimization problem as:

$$\mathcal{L}_m = E_0(V_m) + \int_0^S v(s) [z(s) - w_m(s)] dF(s)$$

or, equivalently:

$$\begin{aligned} \mathcal{L}_m &= \beta E_1(Z) n_m + (\mu_\theta - \beta) n_m \int_0^S w_m(s) dF(s) \\ &\quad - \gamma \frac{n_m(n_m - 1)}{n_m - 2} \left[\int_0^S (w_m(s))^2 dF(s) - \left(\int_0^S w_m(s) dF(s) \right)^2 \right] \\ &\quad + \int_0^S v(s) [z(s) - w_m(s)] dF(s) \end{aligned}$$

where the intermediary is choosing $w_m(s)$ for each state $s \in [0, S]$ taking as given the market size n_m . We restrict attention to $n_m \geq 3$ so that the trading equilibrium in Proposition 1 involves a well-defined equilibrium price for market m .

The first order condition with respect to $w_m(s)$ delivers:

$$v(s) \stackrel{\text{sign}}{=} E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} - w_m(s) \quad (\text{A.4})$$

where $v(s) \geq 0$ and $w_m(s) \leq z(s)$ hold with complementary slackness.

If $v(s) > 0$, then:

$$w_m(s) = z(s)$$

and, invoking (A.4), we need:

$$z(s) < E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1}$$

to confirm $v(s) > 0$.

If $v(s) = 0$, then (A.4) pins down:

$$w_m(s) = E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1}$$

and we need:

$$z(s) \geq E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1}$$

to confirm $w_m(s) \leq z(s)$.

The payoffs of the equilibrium security are therefore:

$$w_m(s) = \begin{cases} z(s) & \text{if } z(s) < E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} \\ E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} & \text{if } z(s) \geq E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} \end{cases}$$

Suppose there exists an $\bar{s}_m \in (0, S)$ solving:

$$z(\bar{s}_m) \equiv E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} \quad (\text{A.5})$$

Then $z'(\cdot) > 0$ implies:

$$E_1(W_m) = \int_0^{\bar{s}_m} z(s) dF(s) + \int_{\bar{s}_m}^S z(\bar{s}_m) dF(s) \quad (\text{A.6})$$

and we can rewrite Eq. (A.5) as:

$$\int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) \equiv \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} \quad (\text{A.7})$$

The left-hand side of Eq. (A.7) is increasing in \bar{s}_m so there will be a unique solution $\bar{s}_m \in (0, S)$ if and only if:

$$z(S) - E_1(Z) > \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} \quad (\text{A.8})$$

The ratio $\frac{n_m - 2}{n_m - 1}$ is increasing in n_m and asymptotes to 1 as $n_m \rightarrow \infty$.

If the parameters satisfy $z(S) - E_1(Z) \in \left[\frac{\mu_\theta - \beta}{4\gamma}, \frac{\mu_\theta - \beta}{2\gamma} \right)$, then Eq. (16) has a unique solution $n_S \in [3, \infty)$. For any $n_m \in [3, n_S)$, condition (A.8) holds and the equilibrium

security is given by Eq. (14) with \bar{s}_m as defined in Eq. (A.5). For any $n_m \in [n_S, \infty)$, condition (A.8) does not hold, meaning that there is no $\bar{s}_m \in (0, S)$ solving Eq. (A.5). The equilibrium security is still given by Eq. (14) but with $\bar{s}_m = S$ instead of Eq. (A.5).

If the parameters satisfy $z(S) - E_1(Z) \geq \frac{\mu_\theta - \beta}{2\gamma}$, then condition (A.8) is true for any $n_m \in [3, \infty)$. The equilibrium security is thus given by Eq. (14) with \bar{s}_m as defined in Eq. (A.5). Condition (A.8) being true for any $n_m \in [3, \infty)$ means that there is no solution $n_S \in [3, \infty)$ to Eq. (16). Assigning $n_S = \infty$ here recovers Eq. (A.5) from Eq. (15) for any $n_m \geq 3$.

If the parameters satisfy $z(S) - E_1(Z) < \frac{\mu_\theta - \beta}{4\gamma}$, then condition (A.8) is false for any $n_m \in [3, \infty)$. The equilibrium security is thus given by Eq. (14) with $\bar{s}_m = S$ for all $n_m \in [3, \infty)$. Assigning $n_S = -\infty$ here recovers $\bar{s}_m = S$ from Eq. (15) for any $n_m \geq 3$.

We have now shown that the solution to the intermediary's F.O.C.s belongs to the family of debt securities: W_m pays the entirety of the underlying asset Z up to some threshold state \bar{s}_m , after which it pays a flat amount that does not vary with the state. A perturbation argument similar to Hébert (2018) can be used to confirm the optimality of debt securities in our environment. We sketch this argument in the main text (see the third paragraph after the statement of Proposition 2) so do not reproduce it here. Instead, we confirm that \bar{s}_m as defined by Eq. (A.7) satisfies the S.O.C. for a maximum in an auxiliary problem where the intermediary chooses a threshold state \tilde{s}_m to maximize his expected profit within the family of debt securities.

The objective function for this auxiliary problem is:

$$\begin{aligned} \mathcal{L}_m^{(A)} = & (\mu_\theta - \beta) \left[z(\tilde{s}_m) - \int_0^{\tilde{s}_m} [z(\tilde{s}_m) - z(s)] dF(s) \right] \\ & - \gamma \frac{n_m - 1}{n_m - 2} \left[\int_0^{\tilde{s}_m} [z(\tilde{s}_m) - z(s)]^2 dF(s) - \left(\int_0^{\tilde{s}_m} [z(\tilde{s}_m) - z(s)] dF(s) \right)^2 \right] \end{aligned}$$

The first derivative with respect to \tilde{s}_m is:

$$\frac{\partial \mathcal{L}_m^{(A)}}{\partial \tilde{s}_m} = \left[\mu_\theta - \beta - 2\gamma \frac{n_m - 1}{n_m - 2} \int_0^{\tilde{s}_m} [z(\tilde{s}_m) - z(s)] dF(s) \right] [1 - F(\tilde{s}_m)] z'(\tilde{s}_m)$$

If $n_m < n_S$, then Eq. (A.7) has a unique interior solution $\bar{s}_m \in (0, S)$, which is also the unique interior solution to $\frac{\partial \mathcal{L}_m^{(A)}}{\partial \bar{s}_m} = 0$. The second derivative evaluated at this solution is:

$$\left. \frac{\partial^2 \mathcal{L}_m^{(A)}}{\partial \bar{s}_m^2} \right|_{\bar{s}_m = \bar{s}_m} = -2\gamma \frac{n_m - 1}{n_m - 2} (z'(\bar{s}_m))^2 F(\bar{s}_m) [1 - F(\bar{s}_m)] < 0$$

where the inequality follows from $\bar{s}_m \in (0, S)$. Eq. (A.7) thus defines a local maximum and, since there are no local minima, the local maximum is also the global maximum.

If $n_m > n_S$, then there is no solution $\bar{s}_m < S$ to Eq. (A.7). The only solution to $\frac{\partial \mathcal{L}_m^{(A)}}{\partial \bar{s}_m} = 0$ is therefore $\tilde{s}_m = S$, in which case the second derivative is:

$$\left. \frac{\partial^2 \mathcal{L}_m^{(A)}}{\partial \bar{s}_m^2} \right|_{\bar{s}_m = S} = - \left[\mu_\theta - \beta - 2\gamma \frac{n_m - 1}{n_m - 2} [z(S) - E_1(Z)] \right] f(S) z'(S)$$

This is negative if and only if $\frac{n_m - 2}{n_m - 1} > \frac{2\gamma}{\mu_\theta - \beta} [z(S) - E_1(Z)]$ or, equivalently, $n_m > n_S$.

Notice that Eq. (A.7) is only defined if $\mu_\theta > \beta$. We now demonstrate that $\mu_\theta > \beta$ is necessary and sufficient for the intermediary's participation constraint to be satisfied. The participation constraint requires that the intermediary's maximized expected profit, as given by $E_0(V_m)$ in Eq. (7) when evaluated at the equilibrium security, must be at least as large as $\beta E_1(Z) \times n_m$, which is what the intermediary could get by consuming n_m units of Z at date $t = 2$ instead of using these units to design the security for market m .

If $n_m \geq n_S$, then the intermediary's maximization problem yields $W_m = Z$ and the participation constraint simplifies to:

$$(\mu_\theta - \beta) E_1(Z) \geq \gamma \frac{n_m - 1}{n_m - 2} \mathcal{V}_1(Z) \tag{A.9}$$

Assume $\mu_\theta > \beta$ so that the left-hand side of (A.9) is positive. The right hand side of (A.9) is decreasing in n_m so (A.9) will hold for all $n_m \geq n_S$ if it holds for $n_m = n_S$. Evaluating (A.9) at the definition of n_S in Eq. (16), we get:

$$2z(S) E_1(Z) \geq E(Z^2) + (E_1(Z))^2$$

which is true because Z has the property $z'(\cdot) > 0$.

If $n_m < n_S$, then $\bar{s}_m \in (0, S)$ is defined by Eq. (A.7). The participation constraint requires:

$$(\mu_\theta - \beta) E_1(W_m) \geq \gamma \frac{n_m - 1}{n_m - 2} \mathcal{V}_1(W_m) \quad (\text{A.10})$$

where $E_1(W_m)$ is given by Eq. (A.6) and:

$$\mathcal{V}_1(W_m) = \int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)]^2 dF(s) - \left(\int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) \right)^2 \quad (\text{A.11})$$

Use Eq. (A.7) to rewrite (A.10) as:

$$2E_1(W_m) \int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) \geq \mathcal{V}_1(W_m)$$

then substitute in for $E_1(W_m)$ and $\mathcal{V}_1(W_m)$ to get:

$$\begin{aligned} & 2z(\bar{s}_m) \frac{1}{F(\bar{s}_m)} \int_0^{\bar{s}_m} z(s) dF(s) + \left(\frac{1}{F(\bar{s}_m)} - 1 \right) \left[(z(\bar{s}_m))^2 - \frac{1}{F(\bar{s}_m)} \int_0^{\bar{s}_m} (z(s))^2 dF(s) \right] \\ & \geq \frac{1}{F(\bar{s}_m)} \int_0^{\bar{s}_m} (z(s))^2 dF(s) + \left(\frac{1}{F(\bar{s}_m)} \int_0^{\bar{s}_m} z(s) dF(s) \right)^2 \end{aligned}$$

which is again true because of $z'(\cdot) > 0$. ■

Proof of Proposition 3

For any $n_m < n_S$, Eq. (15) simplifies to Eq. (A.7) from the proof of Proposition 2.

Differentiating Eq. (A.7) yields:

$$\frac{d\bar{s}_m}{dn_m} = \frac{\mu_\theta - \beta}{2\gamma} \frac{1}{(n_m - 1)^2} \frac{1}{z'(\bar{s}_m) F(\bar{s}_m)} > 0$$

Therefore, $\frac{d\bar{s}_m}{dn_m} > 0$ for any $n_m \in [3, n_S)$ and $\lim_{n_m \rightarrow n_S^-} \frac{d\bar{s}_m}{dn_m} > 0$.

A corollary is that the same properties hold for the mean and variance of the equilibrium security. To see why, differentiate Eq. (A.6) and (A.11) to get:

$$\frac{dE_1(W_m)}{d\bar{s}_m} = z'(\bar{s}_m) [1 - F(\bar{s}_m)]$$

and:

$$\frac{d\mathcal{V}_1(W_m)}{d\bar{s}_m} = 2z'(\bar{s}_m)[1 - F(\bar{s}_m)] \int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s)$$

Both of these derivatives are strictly positive because $n_m < n_S$ implies $\bar{s}_m \in (0, S)$. It then follows immediately that $E_1(W_m)$ and $\mathcal{V}_1(W_m)$ increase with n_m as \bar{s}_m increases with n_m , up until the point where $n_m = n_S$. ■

Proof of Proposition 4

A market structure with one active intermediary and all investors trading in the same market is always stable since there is no other active intermediary to which an investor can deviate. The rest of this proof will therefore focus on symmetric market structures with two or more active intermediaries.

A market structure where each active intermediary gets n investors is stable if and only if:

$$E_0(V^i(n)) > E_0(V^i(n+1))$$

From Eq. (17), the expected profit of an investor in a market of size n is:

$$E_0(V^i(n)) = \frac{\sigma_\theta^2 n - 2 [E_1(W(n))]^2}{2\gamma n - 1 \mathcal{V}_1(W(n))} + \frac{\gamma}{2} \frac{n}{n-2} \mathcal{V}_1(W(n)) \quad (\text{A.12})$$

where we write $E_0(V^i(n))$ to make explicit that we are evaluating the investor's expected profit at the equilibrium security derived in Proposition 2, denoted here by $W(n)$ to make explicit its dependence on the market size n .

We first show that the term $\frac{n-2}{n-1} \frac{[E_1(W(n))]^2}{\mathcal{V}_1(W(n))}$ in Eq. (A.12) is bounded. The ratio $\frac{n-2}{n-1}$ is increasing in n with $\lim_{n \rightarrow \infty} \frac{n-2}{n-1} = 1$. Therefore, we only need to show that $\frac{[E_1(W(n))]^2}{\mathcal{V}_1(W(n))}$ is bounded. To do so, take the derivative with respect to n :

$$\frac{d}{dn} \left(\frac{[E_1(W(n))]^2}{\mathcal{V}_1(W(n))} \right) = \frac{E_1(W(n))}{\mathcal{V}_1(W(n))} \left[2 \frac{dE_1(W(n))}{dn} - \frac{E_1(W(n))}{\mathcal{V}_1(W(n))} \frac{d\mathcal{V}_1(W(n))}{dn} \right]$$

If $n \geq n_S$, then $W(n) = Z$ and this derivative is zero. If instead $n < n_S$, then we can use

the derivatives in the proof of Proposition 3 to write:

$$\frac{d}{dn} \left(\frac{[E_1(W(n))]^2}{\mathcal{V}_1(W(n))} \right) \stackrel{\text{sign}}{=} 1 - \frac{E_1(W(n))}{\mathcal{V}_1(W(n))} \int_0^{\bar{s}} [z(\bar{s}) - z(s)] dF(s)$$

and, with $E_1(W(n))$ as per Eq. (A.6) and $\mathcal{V}_1(W(n))$ as per Eq. (A.11), we get:

$$\frac{d}{dn} \left(\frac{[E_1(W(n))]^2}{\mathcal{V}_1(W(n))} \right) \stackrel{\text{sign}}{=} - \frac{\int_0^{\bar{s}} z(s) [z(\bar{s}) - z(s)] dF(s)}{\mathcal{V}_1(W(n))} < 0$$

We can now conclude:

$$\frac{[E_1(W(n))]^2}{\mathcal{V}_1(W(n))} \leq \frac{[E_1(W(3))]^2}{\mathcal{V}_1(W(3))}$$

where $n = 3$ is the smallest market size for which there can be a well-defined equilibrium price in Eq. (12). If $n_S > 3$, then Eq. (A.7) defines $\bar{s} \in (0, S)$ and hence $E_1(W(3)) \in (0, \infty)$ and $\mathcal{V}_1(W(3)) \in (0, \infty)$. In other words, $\frac{[E_1(W(3))]^2}{\mathcal{V}_1(W(3))}$ is bounded. If instead $n_S = 3$, then $\frac{[E_1(W(3))]^2}{\mathcal{V}_1(W(3))} = \frac{[E_1(Z)]^2}{\mathcal{V}_1(Z)}$ which is also bounded.

Next, we show that the term $\frac{n\mathcal{V}_1(W(n))}{n-2}$ in Eq. (A.12) is decreasing in n if the payoffs of the asset Z satisfy $\frac{z(k) - E_1(Z|s \leq k)}{\sqrt{\mathcal{V}_1(Z|s \leq k)}} < \sqrt{2}, \forall k \in (0, S]$. Taking derivatives:

$$\frac{d}{dn} \left(\frac{n\mathcal{V}_1(W(n))}{n-2} \right) = - \frac{2\mathcal{V}_1(W(n))}{(n-2)^2} + \frac{n}{n-2} \frac{d\mathcal{V}_1(W(n))}{dn}$$

If $n \geq n_S$, then $W(n) = Z$ and this derivative is negative. If instead $n < n_S$, then we can use the derivatives in the proof of Proposition 3 to write:

$$\frac{d}{dn} \left(\frac{n\mathcal{V}_1(W(n))}{n-2} \right) \stackrel{\text{sign}}{=} -\mathcal{V}_1(W(n)) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n(n-2)}{(n-1)^2} \frac{1-F(\bar{s})}{F(\bar{s})} \int_0^{\bar{s}} [z(\bar{s}) - z(s)] dF(s)$$

Using Eq. (A.7) and the expression for $\mathcal{V}_1(W(n))$ in Eq. (A.11), we obtain the following necessary and sufficient condition for $\frac{d}{dn} \left(\frac{n\mathcal{V}_1(W(n))}{n-2} \right) < 0$ when $n < n_S$:

$$\int_0^{\bar{s}} [z(\bar{s}) - z(s)]^2 dF(s) > \frac{1}{F(\bar{s})} \left(1 + \frac{1-F(\bar{s})}{n-1} \right) \left(\int_0^{\bar{s}} [z(\bar{s}) - z(s)] dF(s) \right)^2$$

This rearranges to:

$$\frac{z(\bar{s}) - E_1(Z|s \leq \bar{s})}{\sqrt{\mathcal{V}_1(Z|s \leq \bar{s})}} < \sqrt{\frac{n-1}{1-F(\bar{s})}} \quad (\text{A.13})$$

where:

$$E_1(Z|s \leq \bar{s}) \equiv \frac{1}{F(\bar{s})} \int_0^{\bar{s}} z(s) dF(s)$$

and:

$$\mathcal{V}_1(Z|s \leq \bar{s}) \equiv \frac{1}{F(\bar{s})} \int_0^{\bar{s}} (z(s))^2 dF(s) - \left(\frac{1}{F(\bar{s})} \int_0^{\bar{s}} z(s) dF(s) \right)^2$$

Since $F(\bar{s}) \in (0, 1)$ and $n \geq 3$, a sufficient condition for (A.13) is:

$$\frac{z(k) - E_1(Z|s \leq k)}{\sqrt{\mathcal{V}_1(Z|s \leq k)}} < \sqrt{2}, \forall k \in (0, S]$$

which is the condition in the statement of Proposition 4.

Invoking this condition, we can now conclude that there exists a bound $\bar{\sigma} > 0$ such that $\frac{dE_0(V^i(n))}{dn} < 0$ for all $n \geq 3$ if $\sigma_\theta^2 \leq \bar{\sigma}$.

In words, any symmetric market structure is stable when $\sigma_\theta^2 \leq \bar{\sigma}$. For any integer $n \geq 3$ such that $\frac{N}{n}$ is also an integer, the symmetric market structure involves $\frac{N}{n}$ active intermediaries each getting n investors. For any integer $n \geq 3$ such that $\frac{N}{n}$ is not an integer, we can only consider n if there exist positive integers, M_1 and M_2 , such that $M_1 \times n + M_2 \times (n+1) = N$, in which case the symmetric market structure involves M_1 active intermediaries each getting n investors and M_2 active intermediaries each getting $n+1$ investors.

To see what values of n will be consistent with the existence of such integers, consider an arbitrary total number of active intermediaries M' . If each active intermediary gets n investors, then there are $N - M' \times n$ investors left to be allocated to the M' active intermediaries. For a market structure where each of the M' active intermediaries gets either n or $n+1$ investors, we need $N - M' \times n \geq 0$ (so that no active intermediary gets fewer than n investors) and $N - M' \times n \leq M'$ (so that no active intermediary gets more than $n+1$ investors). In other words, we need $M' \in \left[\frac{N}{1+n}, \frac{N}{n} \right]$. We also need M' to be an integer and hence we need an integer to exist between $\frac{N}{1+n}$ and $\frac{N}{n}$. This implies that we can only consider n such that $\left\lfloor \frac{N}{n+1} \right\rfloor < \left\lfloor \frac{N}{n} \right\rfloor$, where the notation $\lfloor X \rfloor$ means X is rounded down

to the nearest integer. As long as N is not too low, $\lfloor \frac{N}{4} \rfloor < \lfloor \frac{N}{3} \rfloor$ will be satisfied, meaning that there will exist a stable market structure where $M_1 \in \mathbb{N}^+$ active intermediaries get 3 investors each and $M_2 \in \mathbb{N}^0$ active intermediaries get 4 investors each. ■

Proof of Proposition 5

Taking derivatives:

$$G'(n_m) \stackrel{\text{sign}}{=} \frac{E_1(W(n_m))}{(n_m - 1)(n_m - 2)} + 2 \frac{dE_1(W(n_m))}{dn_m} - \frac{E_1(W(n_m))}{\mathcal{V}_1(W(n_m))} \frac{d\mathcal{V}_1(W(n_m))}{dn_m}$$

and:

$$R'(n_m) \stackrel{\text{sign}}{=} - \left(\frac{2\mathcal{V}_1(W(n_m))}{n_m(n_m - 2)} - \frac{d\mathcal{V}_1(W(n_m))}{dn_m} \right)$$

If $n_m > n_S$, then $W(n_m) = Z$ and thus $\frac{dE_1(W(n_m))}{dn_m} = \frac{d\mathcal{V}_1(W(n_m))}{dn_m} = 0$, which further implies $G'(n_m) > 0$ and $R'(n_m) < 0$. If instead $n_m \leq n_S$, then Proposition 2 defines:

$$z(\bar{s}_m) = E_1(W(n_m)) + \frac{\mu_\theta - \beta n_m - 2}{2\gamma} \frac{n_m - 2}{n_m - 1}$$

With $z(s) = z(0) + s^\alpha$ and $f(s) = \frac{1}{S}$, the equilibrium security has:

$$E_1(W(n_m)) = z(0) + \bar{s}_m^\alpha \left(1 - \frac{\alpha}{1 + \alpha} \frac{\bar{s}_m}{S} \right)$$

and:

$$\mathcal{V}_1(W(n_m)) = \frac{\alpha^2}{1 + \alpha} \frac{\bar{s}_m^{1+2\alpha}}{S} \left(\frac{2}{1 + 2\alpha} - \frac{1}{1 + \alpha} \frac{\bar{s}_m}{S} \right)$$

where:

$$\frac{\alpha}{1 + \alpha} \frac{\bar{s}_m^{1+\alpha}}{S} = \frac{\mu_\theta - \beta n_m - 2}{2\gamma} \frac{n_m - 2}{n_m - 1}$$

The total derivatives of $E_1(W(n_m))$ and $\mathcal{V}_1(W(n_m))$ with respect to n_m are therefore:

$$\frac{dE_1(W(n_m))}{dn_m} = \left(1 - \frac{\bar{s}_m}{S} \right) \frac{\mu_\theta - \beta}{2\gamma \frac{\bar{s}_m}{S}} \frac{1}{(n_m - 1)^2}$$

and:

$$\frac{d\mathcal{V}_1(W(n_m))}{dn_m} = \frac{2\alpha}{1 + \alpha} \frac{\bar{s}_m^{1+\alpha}}{S} \frac{dE_1(W(n_m))}{dn_m}$$

Substituting into the expressions for $G'(n_m)$ and $R'(n_m)$, we get:

$$G'(n_m) \stackrel{\text{sign}}{=} \frac{\mu_\theta - \beta}{2\gamma \frac{\bar{s}_m}{S}} \frac{1}{(n_m - 1)^2} \frac{\bar{s}_m^\alpha \left[\frac{2\alpha}{1+2\alpha} + \frac{1-4\alpha}{1+2\alpha} \frac{\bar{s}_m}{S} + \frac{\alpha}{1+\alpha} \left(\frac{\bar{s}_m}{S} \right)^2 \right] - z(0) \left[\frac{2\alpha}{1+2\alpha} - \frac{\bar{s}_m}{S} \right]}{\alpha \bar{s}_m^\alpha \left(\frac{2}{1+2\alpha} - \frac{1}{1+\alpha} \frac{\bar{s}_m}{S} \right)}$$

and:

$$R'(n_m) \stackrel{\text{sign}}{=} - \frac{2\alpha^2}{(1+\alpha)^2} \frac{\bar{s}_m^{1+2\alpha}}{S} \frac{\left[\left(\frac{1+\alpha}{\alpha} \frac{\mu_\theta - \beta}{2\gamma S^\alpha} \frac{n_m - 2}{n_m - 1} \right)^{\frac{1}{1+\alpha}} - \frac{2(1+\alpha) - n_m}{1+2\alpha} \right]}{n_m (n_m - 1) (n_m - 2)}$$

To help establish $R'(n_m) < 0$, notice that $R'(n_m) < 0$ for all $n_m \geq 3$ if and only if $R'(3) < 0$. Therefore, $\left(\frac{1+\alpha}{4\alpha} \frac{\mu_\theta - \beta}{\gamma S^\alpha} \right)^{\frac{1}{1+\alpha}} > \frac{2\alpha - 1}{1+2\alpha}$ is sufficient for $R'(n_m) < 0$. To help establish $G'(n_m) > 0$, define the function $h(x) \equiv \frac{2\alpha}{1+2\alpha} + \frac{1-4\alpha}{1+2\alpha}x + \frac{\alpha}{1+\alpha}x^2$, where $x \in [0, 1]$. Notice $h(0) > 0$. Also notice $h''(x) > 0$ so any solution to $h'(x) = 0$ is a minimum. If $\alpha \in (0, \frac{1}{4}]$, then there is no $x_0 \in [0, 1]$ solving $h'(x_0) = 0$, hence $h(x) > 0$ for all $x \in [0, 1]$. If instead $\alpha > \frac{1}{4}$, then $x_0 = \frac{(1+\alpha)(4\alpha-1)}{2\alpha(1+2\alpha)}$ and $h(x_0) = \frac{7\alpha-1}{4\alpha(1+2\alpha)^2} > 0$, where the inequality follows from $\alpha > \frac{1}{4}$. We again have $h(x) > 0$ for all $x \in [0, 1]$. Notice that $h\left(\frac{\bar{s}_m}{S}\right) > 0$ for all $\frac{\bar{s}_m}{S} \in (0, 1]$ implies $G'(n_m) > 0$ when $z(0) = 0$ so, by continuity, $G'(n_m) > 0$ for any $z(0)$ below some positive upperbound. We can exclude $\bar{s}_m = 0$ when discussing $h\left(\frac{\bar{s}_m}{S}\right)$ since $n_m \geq 3$ implies $\bar{s}_m > 0$. ■

Proof of Proposition 6

Start with the intermediary's expected payoff, $E_0(V_m)$. Substituting $\lambda_m^{-1} \equiv (n_m - 2)$ into Eq. (13):

$$E_0(V_m) = \left[\beta E_1(Z) + (\mu_\theta - \beta) E_1(W(n_m)) - \frac{n_m - 1}{n_m - 2} \gamma \mathcal{V}_1(W(n_m)) \right] \times n_m$$

This expression for $E_0(V_m)$ is increasing in n_m holding constant the security W_m , implying that $E_0(V_m)$ is increasing in n_m for $n_m > n_S$ since the equilibrium security for any $n_m > n_S$ is simply $W_m = Z$. It only remains to check that $E_0(V_m)$ is also increasing in n_m for $n_m \leq n_S$ when evaluated at the equilibrium security $W(n_m)$.

Using the expressions for $E_1(W(n_m))$, $\mathcal{V}_1(W(n_m))$, and \bar{s}_m from the proof of Proposi-

tion 5, we can write:

$$E_0(V_m) = S^\alpha \left[\frac{\beta}{1+\alpha} + \frac{\mu_\theta z(0)}{S^\alpha} + (\mu_\theta - \beta) \left(\frac{\bar{s}_m}{S} \right)^\alpha \left(\frac{1+\alpha}{1+2\alpha} - \frac{\alpha}{2(1+\alpha)} \frac{\bar{s}_m}{S} \right) \right] n_m$$

for $n_m \leq n_S$. It is easy to show that $\left(\frac{\bar{s}_m}{S} \right)^\alpha \left(\frac{1+\alpha}{1+2\alpha} - \frac{\alpha}{2(1+\alpha)} \frac{\bar{s}_m}{S} \right)$ is increasing in $\frac{\bar{s}_m}{S}$ for any $\frac{\bar{s}_m}{S} \in [0, 1]$. We also know from Proposition 3 that \bar{s}_m is increasing in n_m . Therefore, $\frac{dE_0(V_m)}{dn_m} > 0$ for $n_m \leq n_S$.

Turn now to total welfare. Ignore the integered nature of investors for the moment. There are N investors, each getting the expected payoff $E_0(V_m^i)$ in Eq. (17). There are also $\frac{N}{n_m}$ active intermediaries, each getting the expected payoff $E_0(V_m)$ in Eq. (13). Inactive intermediaries receive a payoff of zero. Total (expected) welfare at date $t = 0$ is then:

$$\mathcal{W} = N \times E_0(V_m^i) + \frac{N}{n_m} \times E_0(V_m)$$

where $\frac{1}{n_m} \times E_0(V_m)$ is the expected, per-capita payoff of an active intermediary.

Substituting in Eq. (13) and (17):

$$\mathcal{W} = N \left(\beta E_1(Z) + (\mu_\theta - \beta) E_1(W_m) + \frac{\sigma_\theta^2 n_m - 2 [E_1(W_m)]^2}{2\gamma n_m - 1} \frac{1}{\mathcal{V}_1(W_m)} - \frac{\gamma}{2} \mathcal{V}_1(W_m) \right) \quad (\text{A.14})$$

Notice that the utility investors receive from the risk premium (i.e., compensation for risk term) is outweighed by the negative effect of variance on the price that the intermediary receives.

The expression for \mathcal{W} is increasing in n_m holding constant the security W_m . Therefore, \mathcal{W} is increasing in n_m for $n_m > n_S$ and it only remains to check that \mathcal{W} is also increasing in n_m for $n_m \leq n_S$ when evaluated at the equilibrium security. Using the expressions for $E_1(W(n_m))$, $\mathcal{V}_1(W(n_m))$, and \bar{s}_m from the proof of Proposition 5, we can write:

$$\begin{aligned} \mathcal{W} = & \beta \left(z(0) + \frac{S^\alpha}{1+\alpha} \right) N + z(0) \left[\hat{\mu}_\theta + \frac{1}{\alpha} \frac{\hat{\sigma}_\theta^2}{\hat{\mu}_\theta} \frac{2 \left(1 - \frac{\alpha x}{1+\alpha} \right) + \frac{z(0)}{x^\alpha S^\alpha}}{\frac{2}{1+2\alpha} - \frac{x}{1+\alpha}} \right] \gamma S^\alpha N \\ & + \left[\hat{\mu}_\theta x^\alpha \left(1 - \frac{\alpha x}{1+\alpha} \right) + \frac{1}{\alpha} \frac{\hat{\sigma}_\theta^2}{\hat{\mu}_\theta} \frac{x^\alpha \left(1 - \frac{\alpha x}{1+\alpha} \right)^2}{\frac{2}{1+2\alpha} - \frac{x}{1+\alpha}} - \frac{\alpha^2 x^{1+2\alpha}}{2(1+\alpha)} \left(\frac{2}{1+2\alpha} - \frac{x}{1+\alpha} \right) \right] \gamma S^{2\alpha} N \end{aligned}$$

for $n_m \leq n_S$, where $x \equiv \frac{\bar{s}_m}{S}$, $\hat{\mu}_\theta \equiv \frac{\mu_\theta - \beta}{\gamma S^\alpha}$, and $\hat{\sigma}_\theta \equiv \frac{\sigma_\theta}{\gamma S^\alpha}$. Taking derivatives:

$$\begin{aligned} \frac{d\mathcal{W}}{dx} = & \frac{\hat{\sigma}_\theta^2}{\hat{\mu}_\theta} \frac{\frac{2}{1+\alpha} \frac{1}{1+2\alpha} - \frac{z(0)}{x^{1+\alpha} S^\alpha} \left(\frac{2\alpha}{1+2\alpha} - x \right)}{\left(\frac{2}{1+2\alpha} - \frac{x}{1+\alpha} \right)^2} \frac{\gamma S^\alpha N}{\alpha} z(0) \\ & + \alpha x^{\alpha-1} \left[\frac{\hat{\sigma}_\theta^2}{\hat{\mu}_\theta} \frac{\left(1 - \frac{\alpha x}{1+\alpha} \right) h(x)}{\alpha^2 \left(\frac{2}{1+2\alpha} - \frac{x}{1+\alpha} \right)^2} + \left(\hat{\mu}_\theta - \frac{\alpha x^{1+\alpha}}{1+\alpha} \right) (1-x) \right] \gamma S^{2\alpha} N \end{aligned}$$

with the function $h(x) > 0$ as defined in the proof of Proposition 5. The expression for \bar{s}_m from the same proof implies:

$$\frac{2\alpha x^{1+\alpha}}{1+\alpha} = \hat{\mu}_\theta \frac{n_m - 2}{n_m - 1}$$

and hence:

$$\hat{\mu}_\theta - \frac{\alpha x^{1+\alpha}}{1+\alpha} = \frac{\hat{\mu}_\theta n_m}{2(n_m - 1)} > 0$$

Therefore, the second line in the expression for $\frac{d\mathcal{W}}{dx}$ is positive. If $z(0) = 0$, then it follows immediately that $\frac{d\mathcal{W}}{dx} > 0$. If instead $z(0) > 0$, then the first line in the expression for $\frac{d\mathcal{W}}{dx}$ is positive if and only if:

$$\left[\frac{\alpha(1-x)}{x^{1+\alpha} S^\alpha} - \frac{1}{2x^\alpha S^\alpha} \right] z(0) < \frac{1}{1+\alpha}$$

A sufficient condition is $\frac{\alpha z(0)}{x^{1+\alpha} S^\alpha} < \frac{1}{1+\alpha}$ evaluated at $x = \left(\frac{(1+\alpha)\hat{\mu}_\theta}{4\alpha} \right)^{\frac{1}{1+\alpha}}$, which is the lowest possible x , specifically the x associated with $n_m = 3$. In other words, $z(0) < \frac{\hat{\mu}_\theta S^\alpha}{4\alpha^2}$ is sufficient for $\frac{d\mathcal{W}}{dx} > 0$. The fully constrained planner thus chooses $n_m = N$ and $W_m = Z$ when $z(0)$ is not too large.

Return now to the integer nature of investors. Denote by $\mathcal{W}(n_m)$ the right-hand side of Eq. (A.14), where $W_m \equiv W(n_m)$ is the equilibrium security. In a market structure satisfying condition (18), aggregate welfare is:

$$\mathcal{W} = n \times M_1 \times \frac{\mathcal{W}(n)}{N} + (N - n \times M_1) \times \frac{\mathcal{W}(n+1)}{N} \leq \mathcal{W}(n+1) \leq \mathcal{W}(N)$$

where the inequalities follow from the fact that $\mathcal{W}(n)$ is increasing in n when $z(0)$ is not

too large. Recalling that $\mathcal{W}(N)$ is welfare when all investors trade in one market completes the proof. ■

Proofs for Section 5

1. Higher γ

From the expression for \bar{s}_m in the proof of Proposition 5, it is straightforward to see that \bar{s}_m is decreasing in γ , all else constant. We can also see that $E_1(W_m)$ and $\mathcal{V}_1(W_m)$ are increasing in \bar{s}_m , with no direct dependence on γ . Therefore, $E_1(W_m)$ and $\mathcal{V}_1(W_m)$ are decreasing in γ , for any market size $n_m \leq n_S$.

At $z(0) = 0$ and $\alpha = 1$:

$$\frac{dG(n_m)}{d\gamma} = -\frac{\bar{s}_m}{\gamma} \frac{\sigma_\theta^2}{\mu_\theta - \beta} \frac{(1 - \frac{\bar{s}_m}{2}) \left[\frac{1}{3} + (1 - \bar{s}_m)^2 \right]}{\left(\frac{4}{3} - \bar{s}_m \right)^2} < 0$$

$$\frac{dR(n_m)}{d\gamma} = \frac{\bar{s}_m}{2\gamma} \frac{\mu_\theta - \beta}{4} \frac{n_m}{n_m - 1} \left(\bar{s}_m - \frac{2}{3} \right) > 0$$

$$\frac{dG'(n_m)}{d\gamma} \stackrel{\text{sign}}{=} - \left[\bar{s}_m - \frac{2}{3} + \frac{3}{2} \left(\frac{4}{3} - \bar{s}_m \right)^2 + \frac{\bar{s}_m^2 (1 - \frac{\bar{s}_m}{2})}{(1 - \bar{s}_m) \left(\frac{4}{3} - \bar{s}_m \right)} \right] < 0$$

$$\frac{dR'(n_m)}{d\gamma} = \frac{(\mu_\theta - \beta)^2}{4\gamma^2} \frac{1}{(n_m - 1)^3} \left(1 + \frac{n_m - 4}{6\bar{s}_m} \right) > 0$$

where the last three inequalities follow from $\bar{s}_m \geq \frac{2}{3}$, which is itself implied by Eq. (16) having a finite solution $n_S \geq 3$.

2. Higher $z(0)$

At $\alpha = 1$, the mean and variance of the original asset are:

$$E_1(Z) = z(0) + \frac{1}{2}$$

$$\mathcal{V}_1(Z) = \frac{1}{12}$$

hence $\frac{dE_1(Z)}{dz(0)} > 0$ and $\frac{d\mathcal{V}_1(Z)}{dz(0)} = 0$.

Using the expressions for $E_1(W_m)$, $\mathcal{V}_1(W_m)$, and \bar{s}_m from the proof of Proposition 5:

$$E_1(W_m) = z(0) + \bar{s}_m \left(1 - \frac{\bar{s}_m}{2}\right)$$

$$\mathcal{V}_1(W_m) = \bar{s}_m^3 \left(\frac{1}{3} - \frac{\bar{s}_m}{4}\right)$$

$$\bar{s}_m^2 = \frac{\mu_\theta - \beta}{\gamma} \frac{n_m - 2}{n_m - 1}$$

if $n_m \leq n_S$. It is easy to see $\frac{d\mathcal{V}_1(W_m)}{dz(0)} = \frac{d\bar{s}_m}{dz(0)} = 0$ and $\frac{dE_1(W_m)}{dz(0)} > 0$ given the market size n_m .

Using the expressions for $G(n_m)$ and $R(n_m)$ from the statement of Proposition 5:

$$\frac{dG'(n_m)}{dz(0)} = \frac{2\sigma_\theta^2}{\gamma} \frac{\bar{s}_m^2}{3} + 2 \left(\bar{s}_m - \frac{2}{3}\right) z(0)$$

$$\frac{dR'(n_m)}{dz(0)} = 0$$

Notice that $z(0)$ has no effect on the compensation for risk. In general, the effect of $z(0)$ on the slope of the gains from trade term depends on the value of \bar{s}_m and thus the value of n_m , although $\left.\frac{dG'(n_m)}{dz(0)}\right|_{z(0)=0} > 0$ for the specific case where $z(0)$ is increased from $z(0) = 0$. A sufficient condition for $\frac{dG'(n_m)}{dz(0)} > 0$ regardless of the starting value of $z(0)$ is $\bar{s}_m \geq \frac{2}{3}$, which, from the expression for \bar{s}_m , is equivalent to $\frac{\mu_\theta - \beta}{\gamma} \geq \frac{4}{9} \frac{n_m - 1}{n_m - 2} \geq \frac{8}{9}$, where the second inequality follows from $n_m \geq 3$. Eq. (16) having a finite solution $n_S \geq 3$ requires $\frac{\mu_\theta - \beta}{\gamma} > 1$, hence the sufficient condition for $\frac{dG'(n_m)}{dz(0)} > 0$ is satisfied.

3. Lower κ

Consider the baseline example ($z(0) = 0$, $\alpha = 1$, $S = 1$) but with $z(s) = \kappa s$, where $\kappa > 0$ is a parameter to be varied. The equilibrium security is characterized by:

$$\bar{s}_m^2 = \frac{\mu_\theta - \beta}{\gamma \kappa} \frac{n_m - 2}{n_m - 1} \longrightarrow n_m = 1 + \frac{1}{1 - \frac{\gamma \kappa}{\mu_\theta - \beta} \bar{s}_m^2}$$

$$E_1(W_m) = \kappa \left(1 - \frac{\bar{s}_m}{2}\right) \bar{s}_m$$

$$\mathcal{V}_1(W_m) = \kappa^2 \left(\frac{1}{3} - \frac{\bar{s}_m}{4} \right) \bar{s}_m^3 \longrightarrow \mathcal{V}_1(W_m) = \left(\frac{\mu_\theta - \beta}{\gamma} \frac{n_m - 2}{n_m - 1} \right)^2 \left(\frac{1}{3} \sqrt{\frac{\gamma \kappa}{\mu_\theta - \beta} \frac{n_m - 1}{n_m - 2}} - \frac{1}{4} \right)$$

and the investor's value function, expressed as a function of \bar{s}_m instead of n_m , is:

$$E_0(V^i) = \kappa \left(\mu_\theta - \beta + \frac{1}{2} \frac{\sigma_\theta^2}{\mu_\theta - \beta} \left(\frac{1 - \frac{\bar{s}_m}{2}}{\frac{1}{3} - \frac{\bar{s}_m}{4}} \right)^2 - \frac{\gamma \kappa}{2} \bar{s}_m^2 \right) \left(\frac{1}{3} - \frac{\bar{s}_m}{4} \right) \bar{s}_m$$

where:

$$\frac{dE_0(V^i)}{d\bar{s}_m} \propto \frac{\sigma_\theta^2}{\mu_\theta - \beta} \frac{(1 - \frac{\bar{s}_m}{2}) \left(\frac{1}{3} - \frac{\bar{s}_m}{2} (1 - \frac{\bar{s}_m}{2}) \right)}{\left(\frac{1}{3} - \frac{\bar{s}_m}{4} \right)^2} - \gamma \kappa \left[1 - \bar{s}_m + \frac{n_m - 1}{n_m - 2} \left(\bar{s}_m - \frac{2}{3} \right) \right] \bar{s}_m^2$$

The first term in this derivative is positive for any $\bar{s}_m \in [0, 1]$ and thus any $n_m \geq 3$. The second term is positive for any $n_m \geq 4$; it is also positive for $n_m = 3$ if $\frac{\mu_\theta - \beta}{\gamma \kappa} > \frac{2}{9}$, which is the same condition as in the second part of Proposition 5 when $\kappa = 1$.

Take two assets, Z_1 and Z_2 , with payoffs $z_1(s) = s$ and $z_2(s) = \bar{\kappa}s$ respectively, where $\bar{\kappa} \in (0, 1)$. Consider $\frac{\mu_\theta - \beta}{\gamma} > \frac{2}{9}$ and σ_θ^2 low enough that $\frac{dE_0(V^i)}{d\bar{s}_m} < 0$ for all $n_m \geq 3$ when the underlying asset is Z_2 . Then, the same σ_θ^2 is low enough that $\frac{dE_0(V^i)}{d\bar{s}_m} < 0$ for all $n_m \geq 3$ when the underlying asset is Z_1 . Any symmetric market structure is therefore stable, regardless of whether the underlying asset is Z_1 or Z_2 . Consider specifically n_1 investors per market when the underlying asset is Z_1 and $n_1 + \ell$ investors per market when the underlying asset is Z_2 , where $\ell \geq 1$. Then $\mathcal{V}_1(W_1) > \mathcal{V}_1(W_2)$ if and only if:

$$\bar{\kappa} < \left(\frac{1 + \frac{\ell}{n_1 - 1}}{1 + \frac{\ell}{n_1 - 2}} \right)^3 \left[1 + \frac{3}{4} \frac{\frac{\ell}{(n_1 - 1)(n_1 - 2)} \left(2 + \frac{\ell}{n_1 - 2} + \frac{\ell}{n_1 - 1} \right)}{\left(1 + \frac{\ell}{n_1 - 1} \right)^2 \sqrt{\frac{\gamma}{\mu_\theta - \beta} \frac{n_1 - 1}{n_1 - 2}}} \right]^2$$

For any given values of n_1 and ℓ , there exists a $\bar{\kappa} \in (0, 1)$ such that this inequality holds. ■

Appendix B – Costly Supply

Given n_m , intermediary m chooses a security W_m to supply in market m . He still supplies one unit of W_m per capita but now he chooses the number of units A_m of the asset Z that back the n_m units of W_m . Previously, we had assumed $A_m = n_m$. We now let the intermediary choose A_m at a cost $c(A_m)$, where $c(0) = 0$ and $c'(\cdot) > 0$. To fix ideas, consider $c(A_m) = \frac{\delta}{2}A_m^2$.

Intermediary m 's expected payoff at date $t = 1$ is:

$$V_m = p_m n_m + \beta E_1 (A_m Z - n_m W_m) - \frac{\delta}{2} A_m^2$$

The equilibrium price p_m is still given by Eq. (12) so:

$$E_0 (V_m) = \left[(\mu_\theta - \beta) E_1 (W_m) - \frac{n_m - 1}{n_m - 2} \gamma \mathcal{V}_1 (W_m) \right] n_m + \beta E_1 (Z) A_m - \frac{\delta}{2} A_m^2$$

The Lagrangian for the intermediary's problem can then be written as:

$$\begin{aligned} \mathcal{L} = & (\mu_\theta - \beta) n_m \int_0^S w_m(s) dF(s) \\ & - \frac{\gamma n_m (n_m - 1)}{n_m - 2} \left[\int_0^S (w_m(s))^2 dF(s) - \left(\int_0^S w_m(s) dF(s) \right)^2 \right] \\ & + \beta E_1 (Z) A_m - \frac{\delta}{2} A_m^2 + \int_0^S v(s) [A_m z(s) - n_m w_m(s)] dF(s) + v_A A_m \end{aligned} \quad (\text{B.1})$$

where $v(s) \geq 0$ is the Lagrange multiplier on the feasibility constraint for state s , and $v_A \geq 0$ is the multiplier on $A_m \geq 0$.

The first order condition for $w_m(s)$ is:

$$v(s) = \mu_\theta - \beta - 2\gamma \frac{n_m - 1}{n_m - 2} [w_m(s) - E_1(W_m)] \quad (\text{B.2})$$

where $v(s) \geq 0$ and $A_m z(s) \geq n_m w_m(s)$ hold with complementary slackness. This implies

that the equilibrium security, conditional on n_m , has payoffs:

$$w_m(s) = \begin{cases} \frac{A_m}{n_m} z(s) & \text{if } s < \bar{s}_m \\ \frac{A_m}{n_m} z(\bar{s}_m) & \text{if } s \geq \bar{s}_m \end{cases}$$

where:

$$\bar{s}_m = \arg \min_{k \in [0, S]} \left| z(k) - \frac{n_m}{A_m} \left(E_1(W_m) + \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m - 2}{n_m - 1} \right) \right| \quad (\text{B.3})$$

and:

$$E_1(W_m) = \frac{A_m}{n_m} \left(z(\bar{s}_m) - \int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) \right) \quad (\text{B.4})$$

The first order condition for A_m is:

$$\delta A_m = \beta E_1(Z) + \int_0^S v(s) z(s) dF(s) + v_A \quad (\text{B.5})$$

Using Eq. (B.2) with $E_1(W_m)$ as defined in Eq. (B.4), we can rewrite Eq. (B.5) as:

$$\begin{aligned} \delta A_m &= \mu_\theta E_1(Z) + v_A & (\text{B.6}) \\ &- 2\gamma \frac{n_m - 1}{n_m - 2} \frac{A_m}{n_m} \left[E_1(Z) \int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) - \int_0^{\bar{s}_m} z(s) [z(\bar{s}_m) - z(s)] dF(s) \right] \end{aligned}$$

Consider $A_m > 0$ so that $v_A = 0$:

1. If $\bar{s}_m = S$, then Eq. (B.6) reduces to:

$$A_m = \frac{\mu_\theta E_1(Z)}{\delta + 2\gamma \frac{n_m - 1}{n_m(n_m - 2)} \mathcal{V}_1(Z)}$$

which confirms $A_m > 0$. To confirm that Eq. (B.3) delivers $\bar{s}_m = S$, we need:

$$z(S) - E_1(Z) < \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m}{A_m} \frac{n_m - 2}{n_m - 1}$$

Substituting in the solution for A_m , the condition for $\bar{s}_m = S$ simplifies to:

$$\frac{n_m(n_m - 2)}{n_m - 1} > \frac{2\gamma \mu_\theta [E_1(Z) z(S) - E_1(Z^2)] + \beta \mathcal{V}_1(Z)}{\delta (\mu_\theta - \beta)}$$

The left-hand side of this inequality is increasing in n_m and the right-hand side is positive. Therefore, $\bar{s}_m = S$ if n_m is above some threshold.

2. If the solution to Eq. (B.3) is interior, then \bar{s}_m is defined by:

$$\int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) \equiv \frac{\mu_\theta - \beta}{2\gamma} \frac{n_m}{A_m} \frac{n_m - 2}{n_m - 1} \quad (\text{B.7})$$

and we can use Eq. (B.7) to simplify Eq. (B.6) to:

$$A_m \left(\delta - 2\gamma \frac{n_m - 1}{n_m(n_m - 2)} \int_0^{\bar{s}_m} z(s) [z(\bar{s}_m) - z(s)] dF(s) \right) = \beta E_1(Z) \quad (\text{B.8})$$

Using Eq. (B.8) to substitute A_m out of Eq. (B.7), we can then rewrite Eq. (B.7) as:

$$\frac{\beta E_1(Z)}{\mu_\theta - \beta} \int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) + \int_0^{\bar{s}_m} z(s) [z(\bar{s}_m) - z(s)] dF(s) = \frac{\delta}{2\gamma} \frac{n_m(n_m - 2)}{n_m - 1} \quad (\text{B.9})$$

which implies $\frac{\partial \bar{s}_m}{\partial n_m} > 0$. Notice from Eq. (B.7) that $A_m > 0$ and, to confirm $\bar{s}_m < S$, we need n_m below the threshold that delivered $\bar{s}_m = S$ in the previous bullet.

We have now shown that the key insights of Propositions 2 and 3 continue to hold. If small markets are stable, then debt will be traded in that market structure. If large markets are stable, then equity will be traded in that market structure.

A market structure where all investors trade in the same market is trivially stable, so, as long as N is large, there always exists an equilibrium where investors trade equity in large markets.

It remains to show that the key insights of Proposition 4 also continue to hold. In particular, we want to show that small markets are also stable if heterogeneity in investor preference shocks, σ_θ^2 , is sufficiently low. The investor's expected profit is still given by Eq. (17) so we follow the steps in the proof of Proposition 4. Specifically, if we can show $\frac{d\mathcal{V}_1(W_m)}{dn_m} < \frac{2\mathcal{V}_1(W_m)}{n_m(n_m-2)}$ for any n_m where the equilibrium security is $W_m \neq Z$, then we can conclude that small markets are stable when σ_θ^2 is sufficiently low.

The variance of the equilibrium security derived above is:

$$\mathcal{V}_1(W_m) = \left(\frac{A_m}{n_m}\right)^2 \left[\int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)]^2 dF(s) - \left(\int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) \right)^2 \right]$$

where A_m depends on \bar{s}_m and n_m as per Eq. (B.7) and \bar{s}_m depends on n_m as per Eq. (B.9).

Therefore:

$$\frac{d\mathcal{V}_1(W_m)}{dn_m} = \frac{2\mathcal{V}_1(W_m)}{A_m} \left(\frac{dA_m}{dn_m} - \frac{A_m}{n_m} \right) + \frac{2A_m^2}{n_m^2} [1 - F(\bar{s}_m)] z'(\bar{s}_m) \frac{d\bar{s}_m}{dn_m} \int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s)$$

where:

$$z'(\bar{s}_m) \frac{d\bar{s}_m}{dn_m} = \frac{\frac{\delta}{2\gamma} \frac{n_m^2 - 2n_m + 2}{(n_m - 1)^2}}{\frac{\beta E_1(Z)}{\mu_\theta - \beta} F(\bar{s}_m) + \int_0^{\bar{s}_m} z(s) dF(s)}$$

and:

$$\frac{dA_m}{dn_m} = \frac{A_m}{n_m} \frac{n_m^2 - 2n_m + 2}{(n_m - 1)(n_m - 2)} \left(1 - \frac{\frac{\delta}{\mu_\theta - \beta} F(\bar{s}_m)}{\frac{\beta E_1(Z)}{\mu_\theta - \beta} F(\bar{s}_m) + \int_0^{\bar{s}_m} z(s) dF(s)} \right)$$

The condition we want to check, $\frac{d\mathcal{V}_1(W_m)}{dn_m} < \frac{2\mathcal{V}_1(W_m)}{n_m(n_m - 2)}$, simplifies to:

$$\begin{aligned} & [1 - F(\bar{s}_m)] z'(\bar{s}_m) \frac{d\bar{s}_m}{dn_m} \int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) \\ & < \frac{1}{n_m} \left[\frac{n_m - 1}{n_m - 2} - \frac{n_m}{A_m} \frac{dA_m}{dn_m} \right] \left[\int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)]^2 dF(s) - \left(\int_0^{\bar{s}_m} [z(\bar{s}_m) - z(s)] dF(s) \right)^2 \right] \end{aligned}$$

\Leftrightarrow

$$1 + [1 - F(\bar{s}_m)] \frac{[z(\bar{s}_m) - E_1(Z|s \leq \bar{s}_m)]^2}{\mathcal{V}_1(Z|s \leq \bar{s}_m)} < \frac{\frac{\delta}{2\gamma} \frac{n_m(n_m - 2)}{n_m - 1} \frac{1}{F(\bar{s}_m)} (n_m^2 - 2n_m + 2)}{\left[\frac{\beta E_1(Z)}{\mu_\theta - \beta} + E_1(Z|s \leq \bar{s}_m) \right] [z(\bar{s}_m) - E_1(Z|s \leq \bar{s}_m)]}$$

\Leftrightarrow

$$\frac{[1 - F(\bar{s}_m)] [z(\bar{s}_m) - E_1(Z|s \leq \bar{s}_m)]^2}{(n_m - 1)^2 \mathcal{V}_1(Z|s \leq \bar{s}_m)} < 1 - \frac{\frac{n_m^2 - 2n_m + 2}{(n_m - 1)^2} \mathcal{V}_1(Z|s \leq \bar{s}_m)}{\left[\frac{\beta E_1(Z)}{\mu_\theta - \beta} + E_1(Z|s \leq \bar{s}_m) \right] [z(\bar{s}_m) - E_1(Z|s \leq \bar{s}_m)]}$$

It follows from $z'(\cdot) > 0$ that $E_1(Z) \geq E_1(Z|s \leq \bar{s}_m)$ and $[z(\bar{s}_m) - E_1(Z|s \leq \bar{s}_m)] >$

$\frac{\mathcal{V}_1(Z|s \leq \bar{s}_m)}{E_1(Z|s \leq \bar{s}_m)}$, so a sufficient condition for $\frac{d\mathcal{V}_1(W_m)}{dn_m} < \frac{2\mathcal{V}_1(W_m)}{n_m(n_m-2)}$ is:

$$\frac{[z(\bar{s}_m) - E_1(Z|s \leq \bar{s}_m)]^2}{\mathcal{V}_1(Z|s \leq \bar{s}_m)} < (n_m^2 - 2n_m + 2) \frac{\beta}{\mu_\theta} - 1$$

The right-hand side is increasing in n_m so, with $n_m \geq 3$, it will be enough to have:

$$\frac{z(k) - E_1(Z|s \leq k)}{\sqrt{\mathcal{V}_1(Z|s \leq k)}} < \sqrt{\frac{5\beta}{\mu_\theta}} - 1, \forall k \in (0, S]$$

with $\beta > \frac{\mu_\theta}{5}$.

Appendix C – Alternative Timing

Suppose the timing is such that intermediaries post securities first, then investors choose markets. Market choice is still made before the realization of investor preference shocks, but now intermediaries can compete for investors through security design. By posting securities first, we mean that the intermediary commits to a particular payoff profile before investors choose their markets. The intermediary is rational so his security design problem will take into account the best responses of investors. However, the intermediary cannot post a security whose payoff profile is contingent on the number of investors who show up. That would constitute a customized contract, not a standardized contract. The focus of our paper is on standardized contracts.

Consider two intermediaries, 1 and 2. Intermediary 1 offers a security W_1 and attracts n_1 investors. Intermediary 2 offers a security W_2 and attracts $N - n_1$ investors.

The expected value to investor i of trading W_m in a market of size n_m is still given by $E_0(V_m^i)$ in Eq. (17). In the extreme case of $\sigma_\theta^2 = 0$:

$$E_0(V_m^i) = \frac{\gamma}{2} \frac{n_m}{n_m - 2} \mathcal{V}_1(W_m) \quad (\text{C.1})$$

By a continuity argument, all results derived under $\sigma_\theta^2 = 0$ will extend to $\sigma_\theta^2 \in (0, \bar{\sigma})$, where $\bar{\sigma}$ is some positive upperbound.

Notice that $\frac{n_m}{n_m - 2}$ in Eq. (C.1) is decreasing in n_m . Also recall that W_m is no longer responsive to n_m at the stage where investors choose their markets. Eq. (C.1) says that investors want a more variable security when σ_θ^2 is low. This is because the trading equilibrium delivers a low enough price (or, equivalently, a high enough risk premium) to compensate them for taking the risk. Investors also want to take this risk in very small markets, reflecting the fact that the risk premium increases with an individual investor's price impact.

Given the securities W_1 and W_2 , investors will move around until they are indifferent between the two intermediaries. We abstract from the integer nature of investors here to avoid unnecessary algebra. The best response of investors then yields a market structure

characterized by n_1^* , where n_1^* solves:

$$\frac{n_1^*}{n_1^* - 2} \mathcal{V}_1(W_1) = \frac{N - n_1^*}{N - n_1^* - 2} \mathcal{V}_1(W_2) \quad (\text{C.2})$$

Eq. (C.2) defines n_1^* as a function of $\frac{\mathcal{V}_1(W_1)}{\mathcal{V}_1(W_2)}$. Differentiate Eq. (C.2) to get:

$$\frac{dn_1^*}{d\mathcal{V}_1(W_1)} = \frac{n_1^*}{2} \frac{1}{\frac{1}{n_1^* - 2} + \frac{1}{\frac{n_1^*}{(N - n_1^* - 2)(N - n_1^*)}}} \frac{1}{\mathcal{V}_1(W_1)}$$

This derivative is positive. If intermediary 1 posts a more variable security than intermediary 2, then intermediary 1 will attract more investors.

Each intermediary seeks to maximize his expected profit subject to a state-by-state feasibility constraint on the payoffs of the security he designs. He still offers one unit of the security to each investor in his market and, as in Appendix B, pays a cost to procure the assets that back this security. The Lagrangian for intermediary 1's problem is thus given by Eq. (B.1) but with $n_1 = n_1^*$, where n_1^* depends on W_1 as per Eq. (C.2). The choice variables are the payoffs $w_1(s)$ for each state $s \in [0, S]$ and the number of units A_1 of Z that will back the n_1^* units of W_1 .

The first order condition for $w_1(s)$ is:

$$\begin{aligned} v(s) = & \mu_\theta - \beta - \frac{\gamma}{n_1^* - 2} \left(2(n_1^* - 1) + \frac{(n_1^*)^2 - 4n_1^* + 2}{1 + \frac{n_1^*(n_1^* - 2)}{(N - n_1^* - 2)(N - n_1^*)}} \right) [w_1(s) - E_1(W_1)] \\ & + \frac{1}{\frac{1}{n_1^* - 2} + \frac{n_1^*}{(N - n_1^* - 2)(N - n_1^*)}} \frac{w_1(s) - E_1(W_1)}{\mathcal{V}_1(W_1)} \left[(\mu_\theta - \beta) E_1(W_1) - \int_0^S v(s) w_1(s) dF(s) \right] \end{aligned}$$

Multiply both sides by $w_1(s)$ then integrate over $s \in [0, S]$ to isolate:

$$\int_0^S v(s) w_1(s) dF(s) = (\mu_\theta - \beta) E_1(W_1) - \frac{\gamma}{n_1^* - 2} \left(\frac{2(n_1^* - 1) + \frac{(n_1^*)^2 - 4n_1^* + 2}{1 + \frac{n_1^*(n_1^* - 2)}{(N - n_1^* - 2)(N - n_1^*)}}}{1 + \frac{1}{\frac{1}{n_1^* - 2} + \frac{n_1^*}{(N - n_1^* - 2)(N - n_1^*)}}} \right) \mathcal{V}_1(W_1)$$

We can now rewrite the first order condition for $w_1(s)$ as:

$$v(s) = \mu_\theta - \beta - \frac{\gamma n_1^*}{n_1^* - 2} \frac{n_1^* - 2 + \frac{2(n_1^* - 1)(n_1^* - 2)}{(N - n_1^* - 2)(N - n_1^*)}}{n_1^* - 1 + \frac{n_1^*(n_1^* - 2)}{(N - n_1^* - 2)(N - n_1^*)}} [w_1(s) - E_1(W_1)] \quad (\text{C.3})$$

The first order condition for A_1 still takes the form of (B.5).

In a symmetric equilibrium, both intermediaries offer the same security W . Eq. (C.2) implies $n_1^* = \frac{N}{2}$ which, when substituted into Eq. (C.3), implies:

$$v(s) = \mu_\theta - \beta - \gamma \frac{N^2 - 8}{N(N - 4)} [w(s) - E_1(W)]$$

for each $s \in [0, S]$. Therefore, the security that prevails in a symmetric equilibrium has payoffs:

$$w(s) = \begin{cases} \frac{2A}{N} z(s) & \text{if } s < \bar{s} \\ \frac{2A}{N} z(\bar{s}) & \text{if } s \geq \bar{s} \end{cases}$$

where the threshold $\bar{s} \in [0, S]$ is defined by:

$$\bar{s} = \arg \min_{k \in [0, S]} \left| z(k) - \frac{N}{2A} \left(E_1(W) + \frac{\mu_\theta - \beta}{\gamma} \frac{N(N - 4)}{N^2 - 8} \right) \right| \quad (\text{C.4})$$

and A solves:

$$A = \frac{\beta E_1(Z) + (\mu_\theta - \beta) \int_0^{\bar{s}} z(s) dF(s)}{\delta + \frac{2\gamma(N^2 - 8)}{N^2(N - 4)} \left[\int_0^{\bar{s}} (z(s))^2 dF(s) - \left(\int_0^{\bar{s}} z(s) dF(s) \right)^2 - \int_0^{\bar{s}} z(s) dF(s) \int_{\bar{s}}^S z(\bar{s}) dF(s) \right]} \quad (\text{C.5})$$

If the solution to Eq. (C.4) is interior, we can combine Eq. (C.4) and (C.5) to get:

$$\frac{\beta E_1(Z)}{\mu_\theta - \beta} \int_0^{\bar{s}} [z(\bar{s}) - z(s)] dF(s) + \int_0^{\bar{s}} z(s) [z(\bar{s}) - z(s)] dF(s) = \frac{\delta}{2\gamma} \frac{N^2(N - 4)}{N^2 - 8}$$

We then need:

$$\delta < \frac{2\gamma}{\mu_\theta - \beta} [\mu_\theta [z(S) E_1(Z) - E_1(Z^2)] + \beta \mathcal{V}_1(Z)] \frac{N^2 - 8}{N^2(N - 4)}$$

for the solution to indeed be interior, in which case:

$$\frac{d\bar{s}}{dN} = \frac{\delta}{2\gamma} \frac{N(N^3 - 24N + 64)}{(N^2 - 8)^2} \frac{1}{z'(\bar{s}) \int_0^{\bar{s}} \left[z(s) + \frac{\beta E_1(Z)}{\mu_\theta - \beta} \right] dF(s)} > 0$$

where the inequality follows from $N \geq 6$ to ensure $\frac{N}{2} \geq 3$. Therefore, the alternative timing considered here does not change the result that debt is traded in smaller markets than equity.